Università di Pisa
Dipartimento di Fisica

The quantum nature of light

photon statistics and photon anti-bunching

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Outline of the presentation

- **Photon counting and photon statistics**

  Classification of light according to its statistics

- **Photon coincidence experiments and correlation function**

  Classification of light based on $g^{(2)}(\tau)$

- ***Relation between the two classifications***

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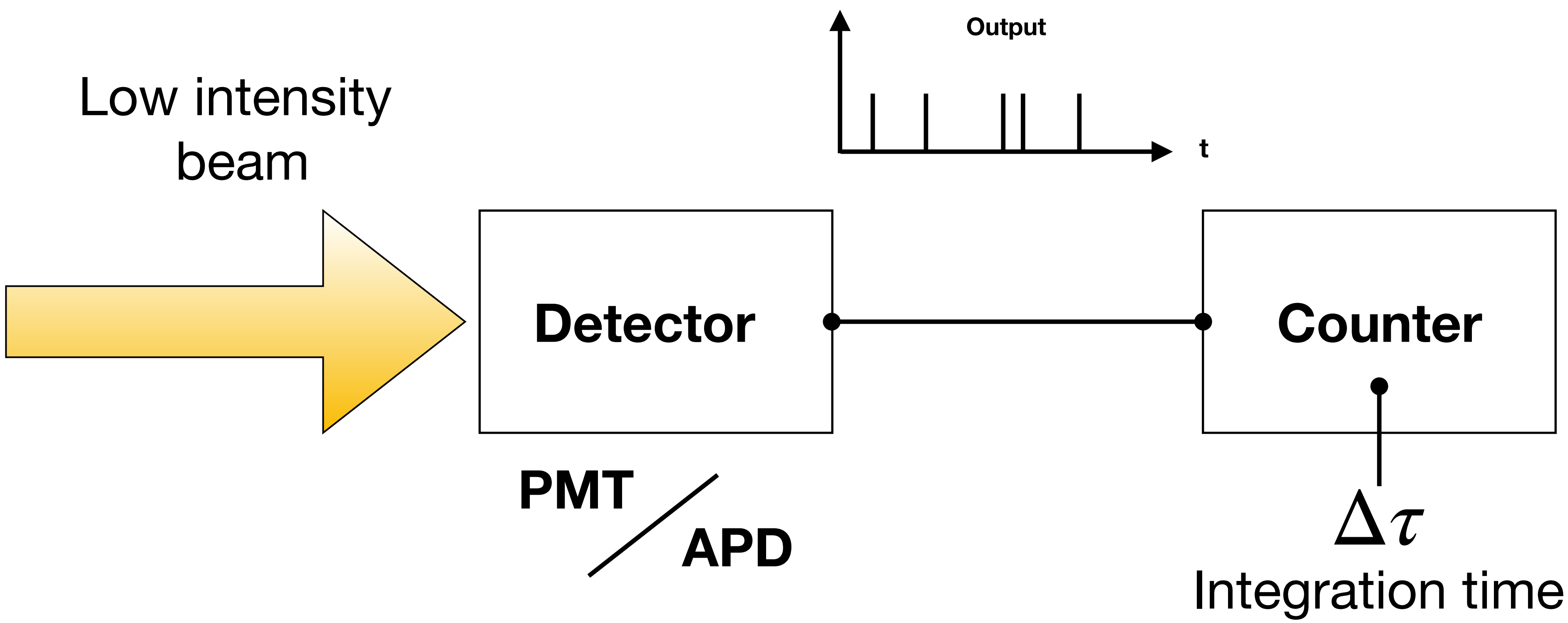
We are interested in photon counting experiments

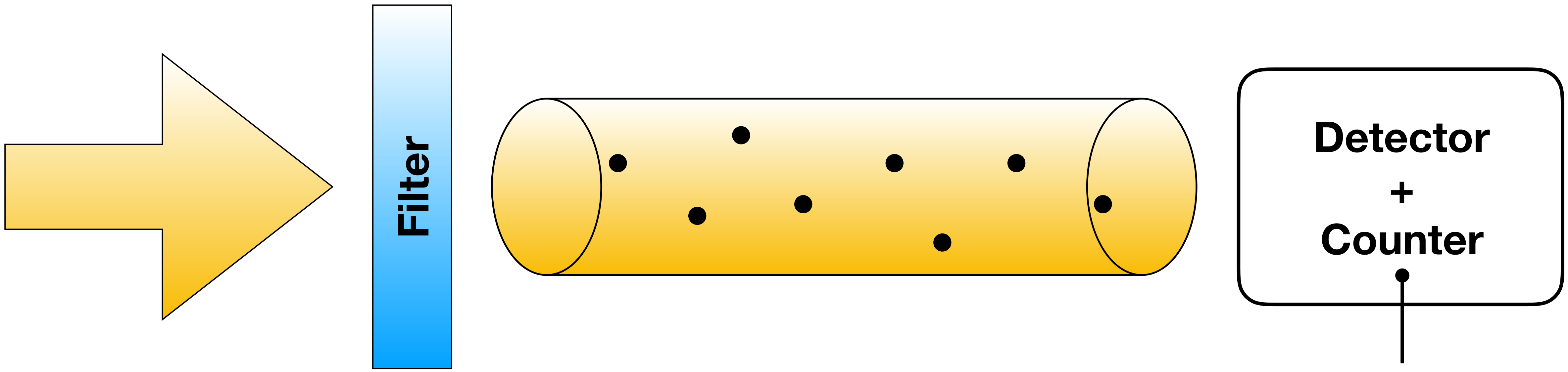


Intensity of the beam → **Average count rate**

Intrinsic photon statistics $\neq$ **Count rate**

The detection process has itself a statistical nature!

A one second long beam of laser light contains ~3 billions of photons



He:Ne Laser

$\lambda = 633$ **nm**

$P = 1$ **mW**

Attenuated beam

$P = 1$ **nW**

Integration time

$\Delta\tau = 1$ **ns**

Average number of counts

$$\bar{n}(\Delta\tau) = \eta \frac{P}{\hbar\omega} \Delta\tau \approx 3.1 \text{ photons}$$

photon flux Φ

In the lab…

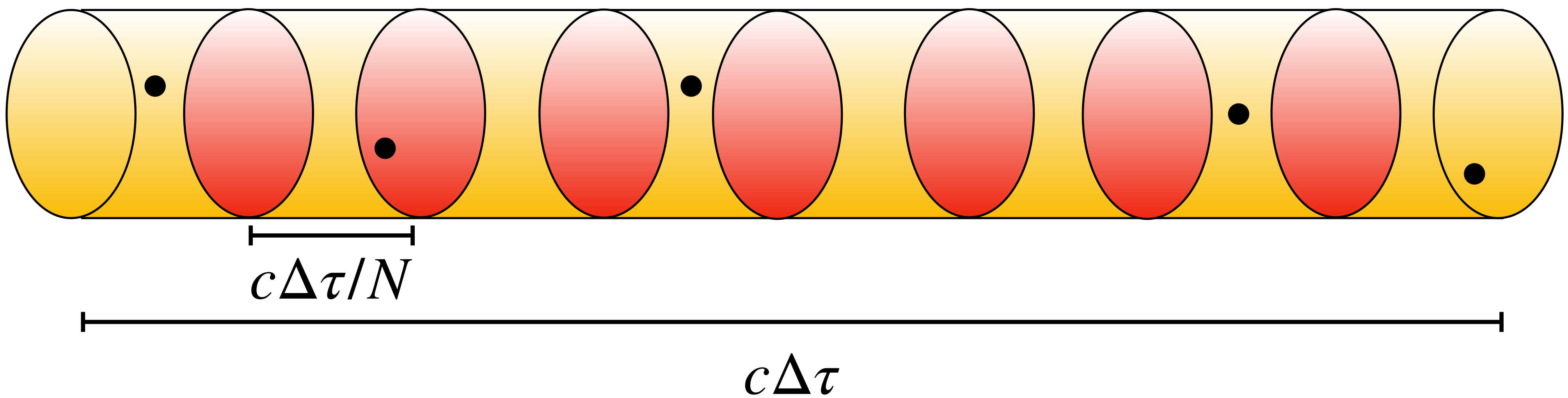
1,6,3,1,2,2,4,4,3,2,4,3,1,3,5,0,1,4,1,1,6,3,4,6,3,2,4,0,3,4,5

A constant monocromatic light beam has a Poissonian statistics

$$E(x,t) = E_0 \sin(kx - \omega t + \phi)$$

Resonably good approximation for a single mode laser

Power of the beam is constant → $\bar{n}(\Delta\tau)$ is well defined



Probability of finding a photon in a subsegment

$$p = \frac{\bar{n}}{N} \qquad N \gg \bar{n}$$

→

Probability of finding n photon in $c\Delta\tau$

$$P(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}$$

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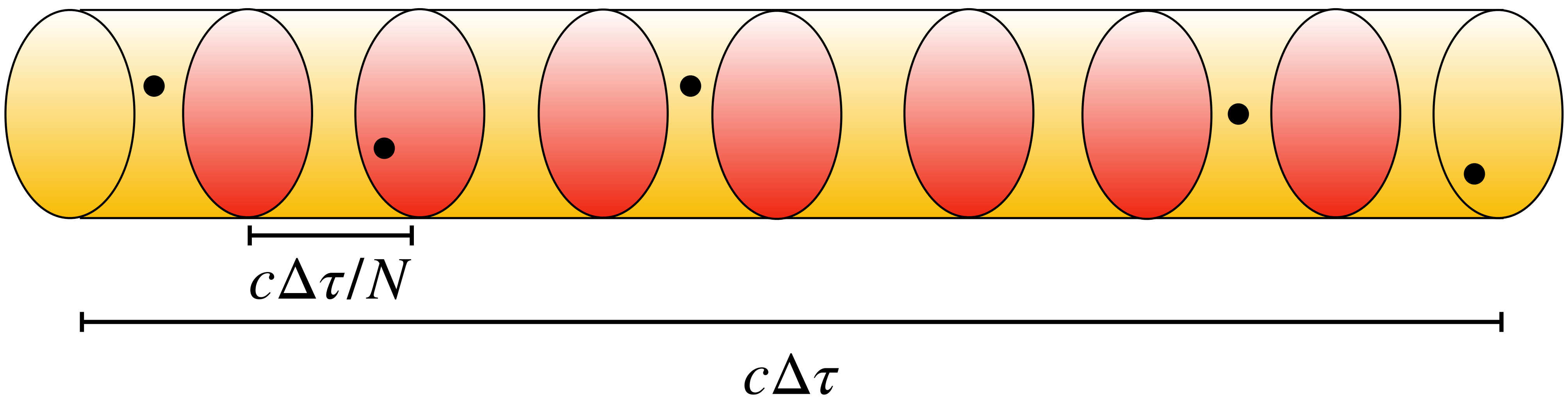


Probability of finding a photon in a subsegment

$$p = \frac{\bar{n}}{N} \qquad N \gg \bar{n}$$

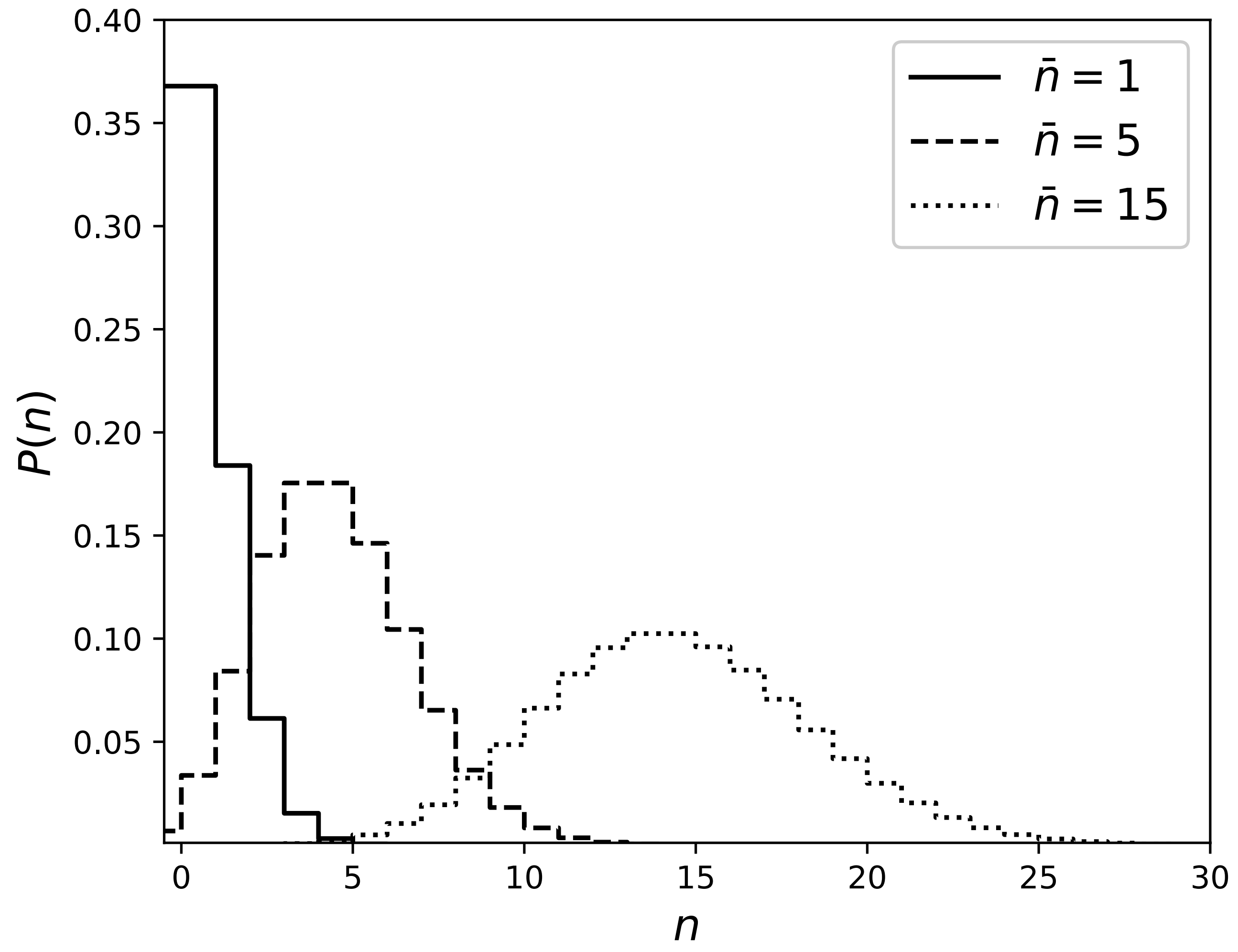
→

Probability of finding n photon in $c\Delta\tau$

$$P(n) = \frac{\bar{n}!}{n!} e^{-\bar{n}}$$

in the $N \to \infty$ limit

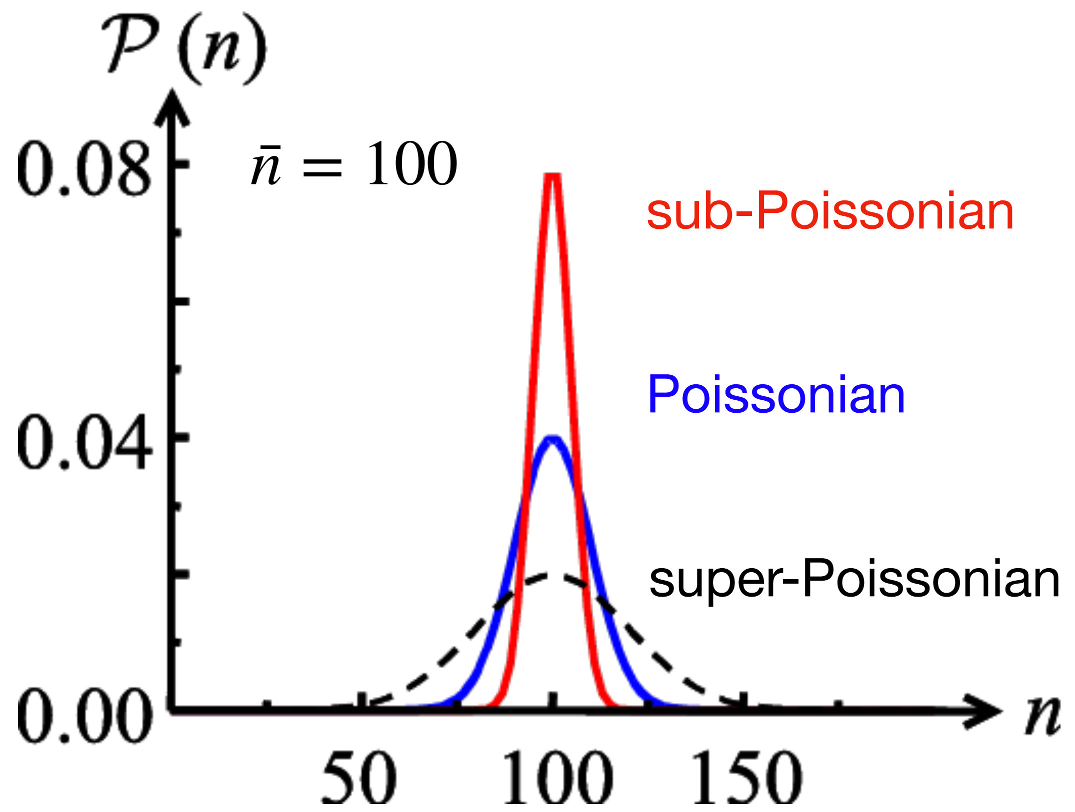
The variance provides a benchmark for classifying different types of light



The distribution is uniquely characterised by the **mean value**

It has variance

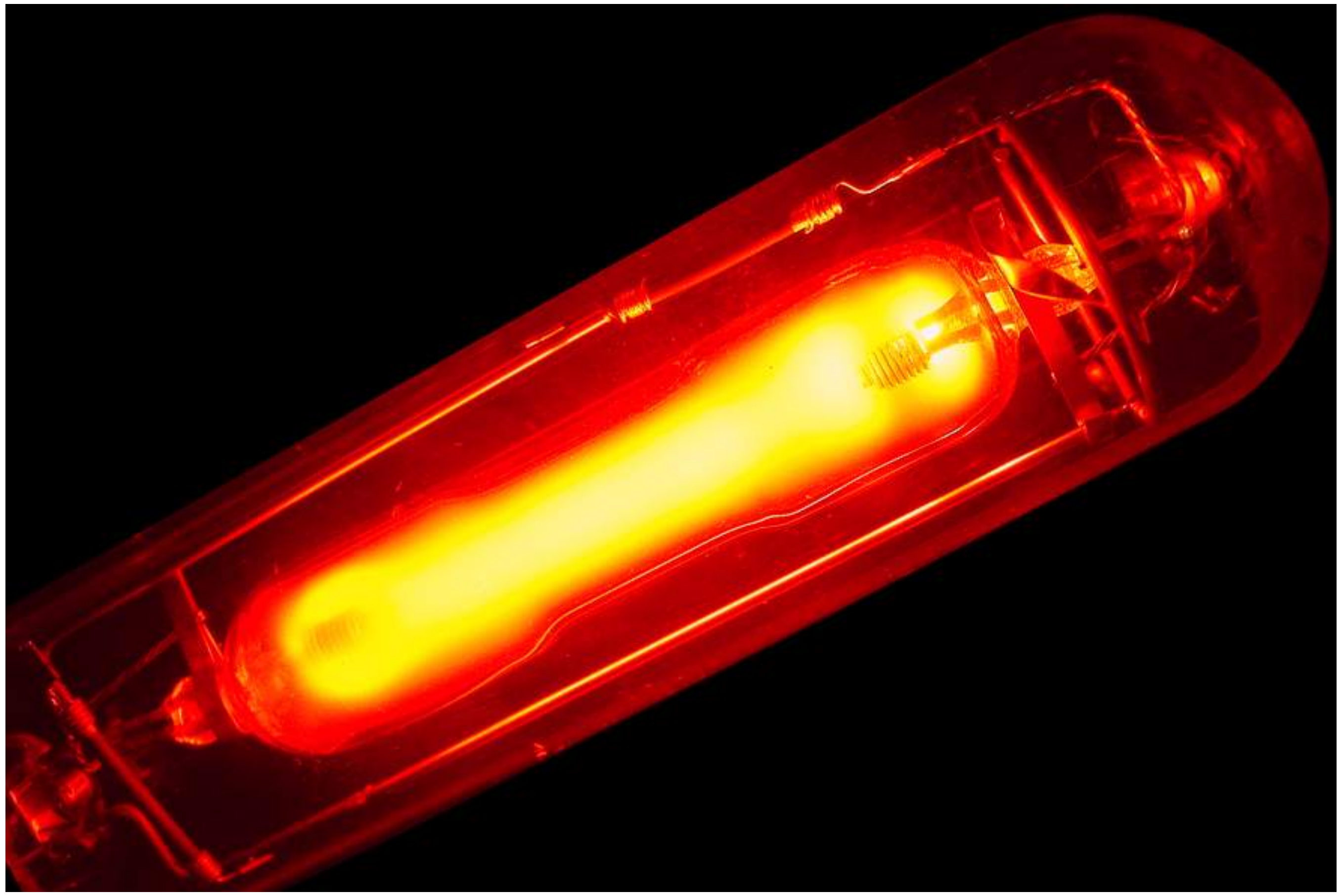
$$Var(n) = \langle \Delta n^2 \rangle = \langle (n - \bar{n})^2 \rangle = \bar{n}$$



- **Poissonian statistics:** $\Delta n^2 = \bar{n}$
- **super-Poissonian statistics:** $\Delta n^2 > \bar{n}$
- **sub-Poissonian statistics:** $\Delta n^2 < \bar{n}$

Chaotic light has a super-Poissonian statistics

Discharge lamp



- **Natural broadening**
- **Collisional broadening**
- **Doppler broadening**

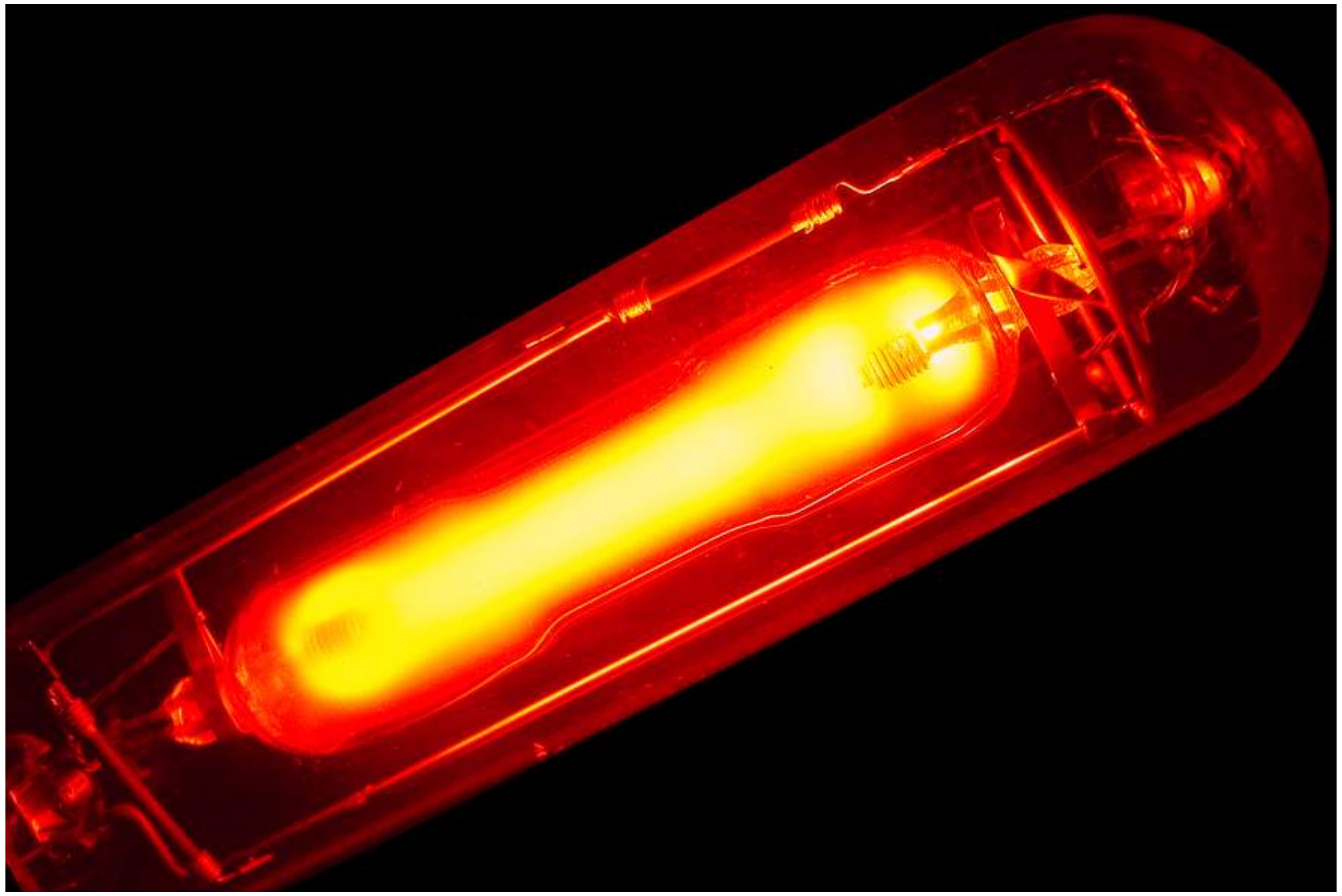
Existence of a **coherence time τ_c**

partially coherent light
not perfectly monochromatic,
slightly fluctuating intensity
on a τ_c time-scale

?

Chaotic light has a super-Poissonian statistics

Discharge lamp



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Existence of a **coherence time τ_c**

partially coherent light
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slightly fluctuating intensity
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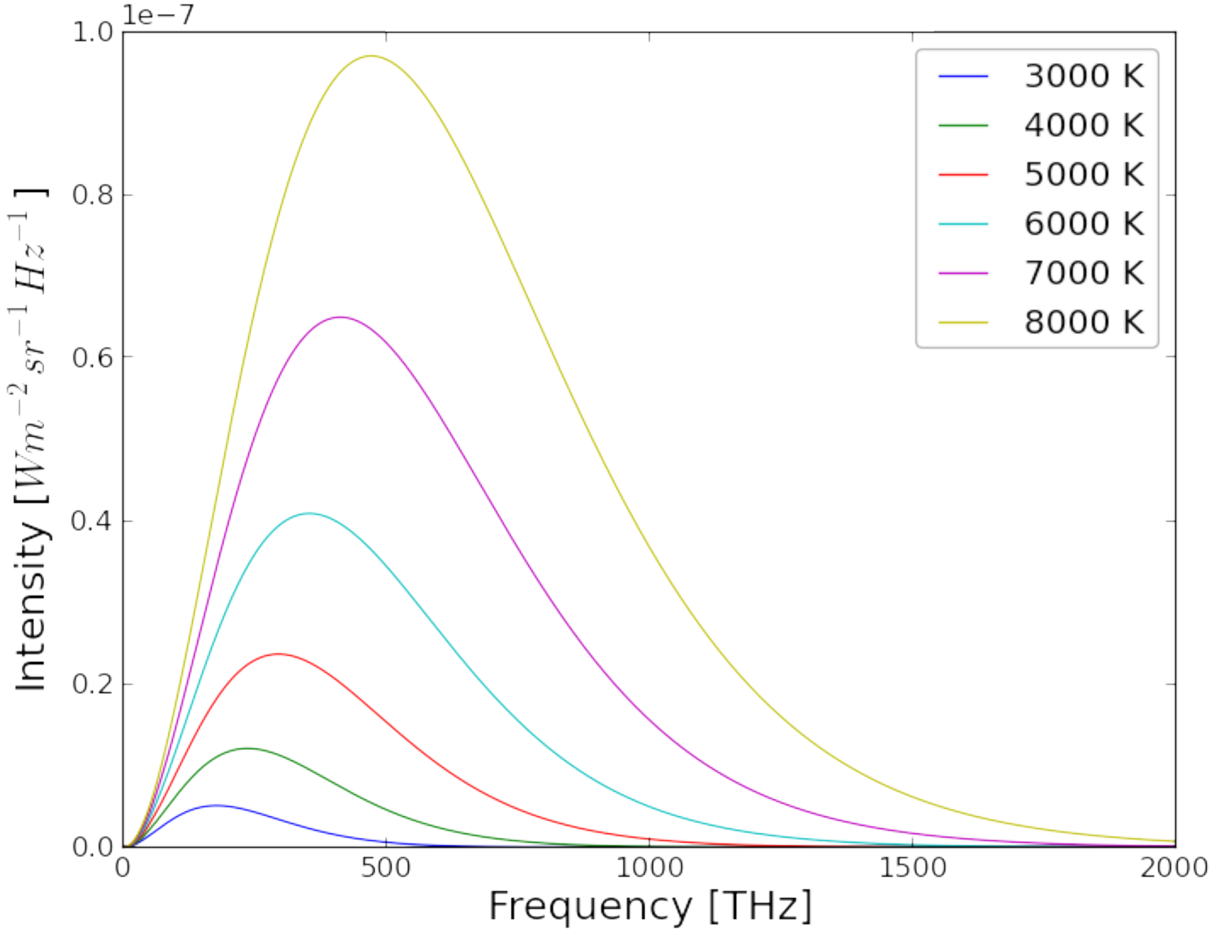
photon count rate

$$W(\Delta\tau) = \int_t^{t+\Delta\tau} \eta\Phi(t')dt'$$

$$(\Delta n)^2 = \langle W(\Delta\tau)\rangle + \langle \Delta W(\Delta\tau)^2\rangle$$

$$\langle W(\Delta\tau)\rangle = \bar{n}$$

A sigle mode of black-body radiation has a super-Poissonian statistics



Plank's law

$$\rho(\omega, T) = \frac{\hbar\omega^3}{\pi^2 c^3} \frac{1}{\exp(\hbar\omega/k_B T) - 1} d\omega$$

single mode
occupation number probability

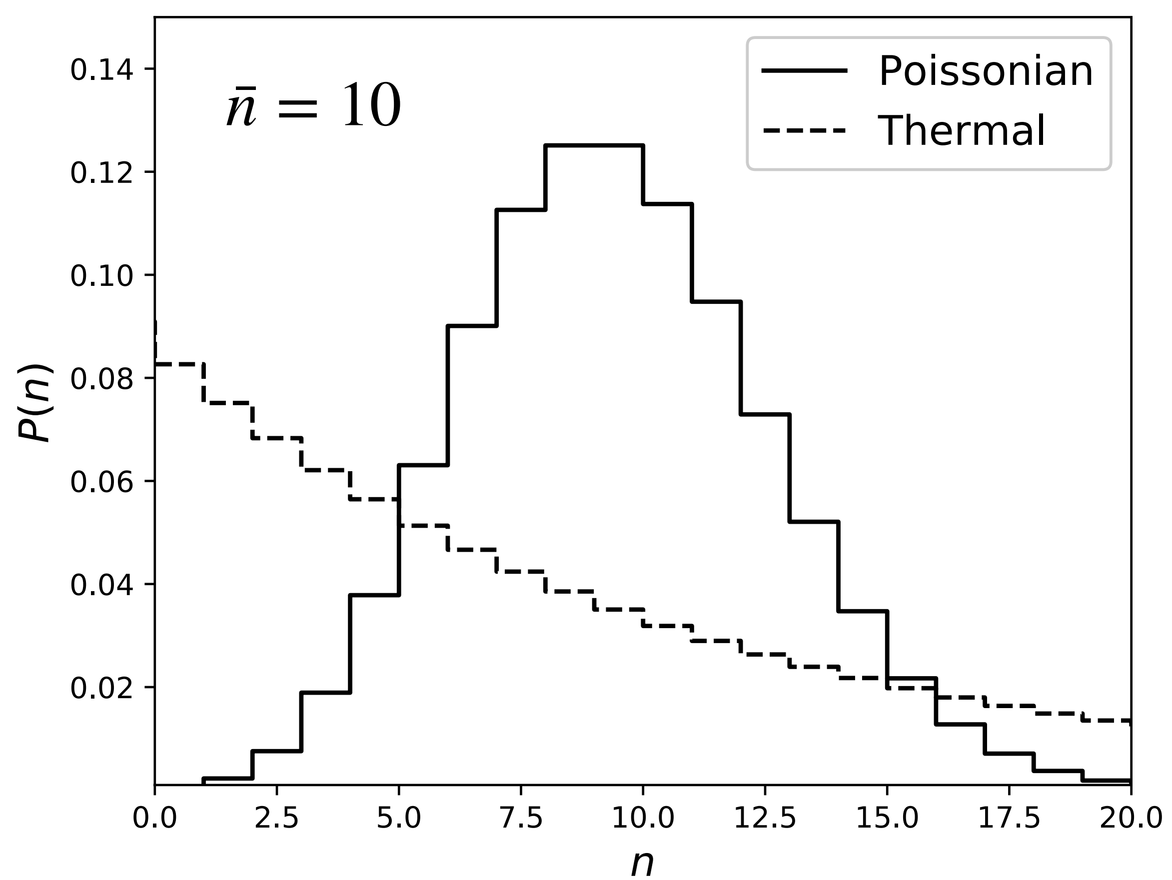
$$P_\omega(n) = \frac{\exp(-n\hbar\omega/k_B T)}{\sum_{n=0}^{\infty} \exp(-n\hbar\omega/k_B T)}$$

Mean

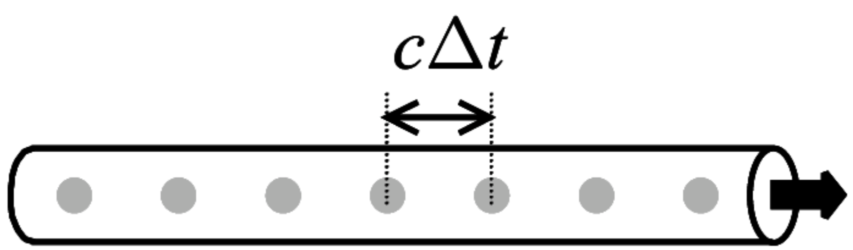
$$\bar{n} = \frac{1}{\exp(\hbar\omega/k_B T) - 1}$$

Variance

$$(\Delta n)^2 = \bar{n} + \bar{n}^2$$

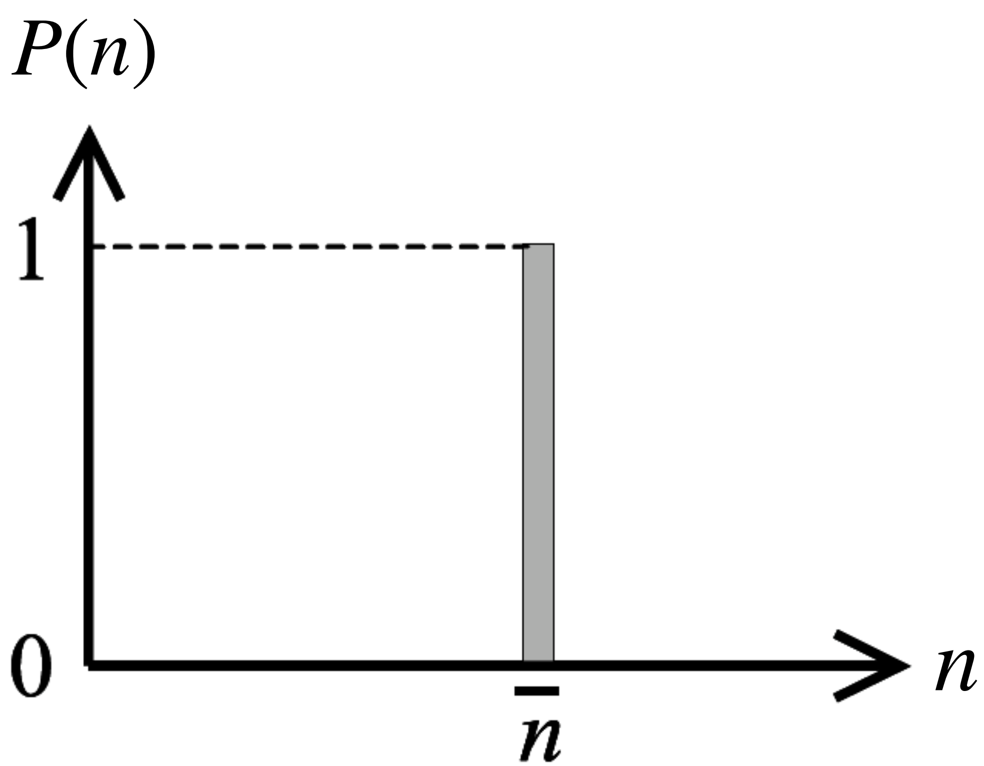


Sub-Poissonian light has to be more stable than a perfectly coherent light



Photon count

$$N = Int\left(\eta\frac{\Delta\tau}{\Delta t}\right) = \bar{n}$$

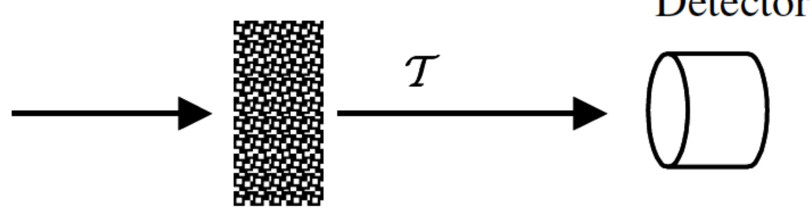


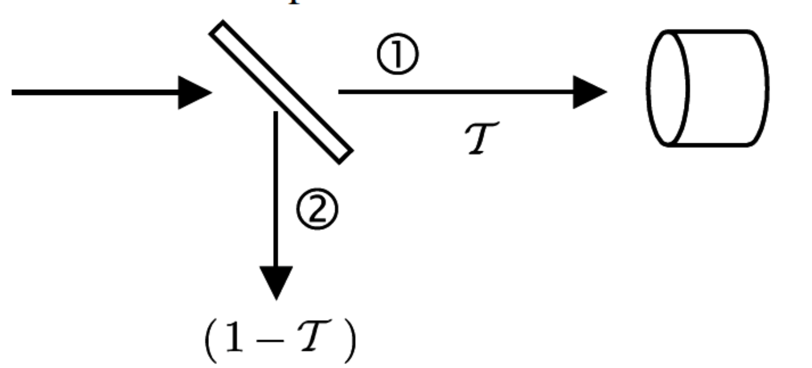
Variance

$$(\Delta n)^2 = 0$$

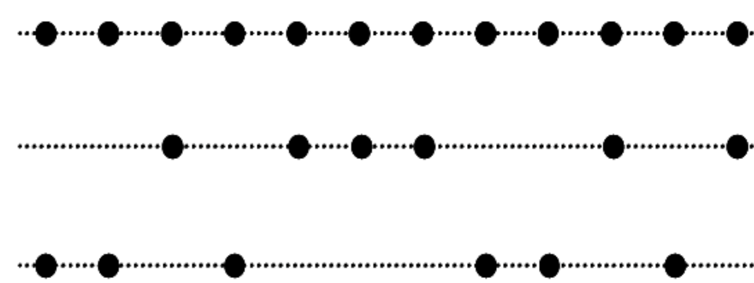
no classical counterpart

The observation of sub-Poissonian light is a direct evidence of the quantum nature of light





- avoid optical losses
- high efficiency detector

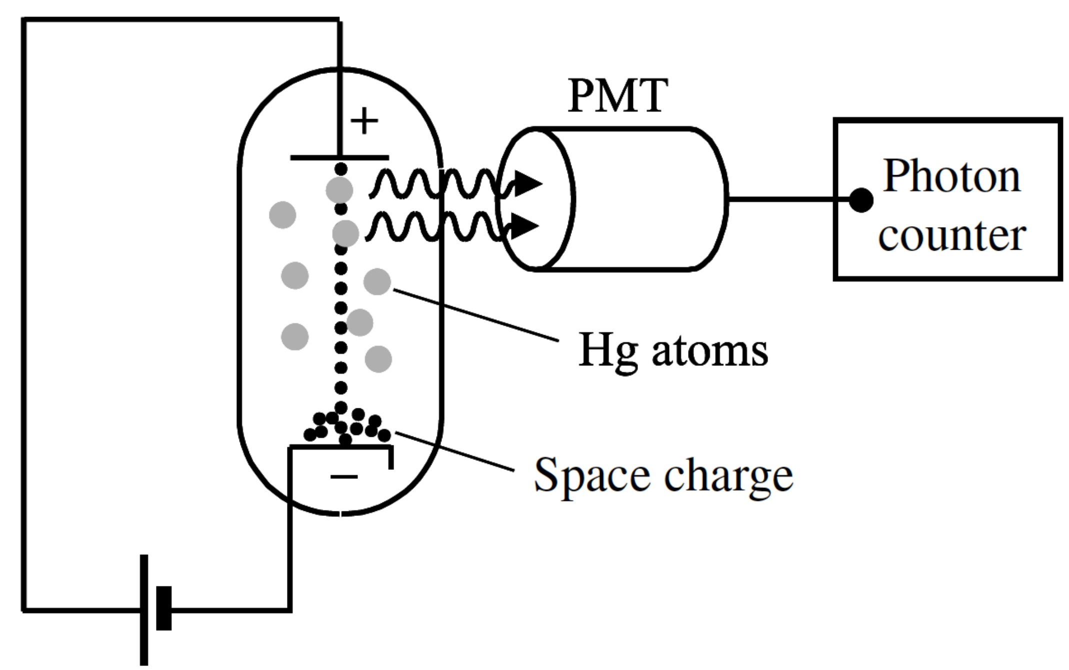


Use a sub-Poissonian source to generate sub-Poissonian light

M.C. Teich and B. E. A. Saleh, J. Opt. Soc. Am. B 2, 275 (1985)

the observed variance was only 0.16% less than a Possonian

- inefficiency of emission (25%)
- imperfect optics transition (83%)
- imperfect photon collection (10%)
- low efficiency of the detector (15%)



M.C. Teich and B. E. A. Saleh, J. Opt. Soc. Am. B 2, 275 (1985)

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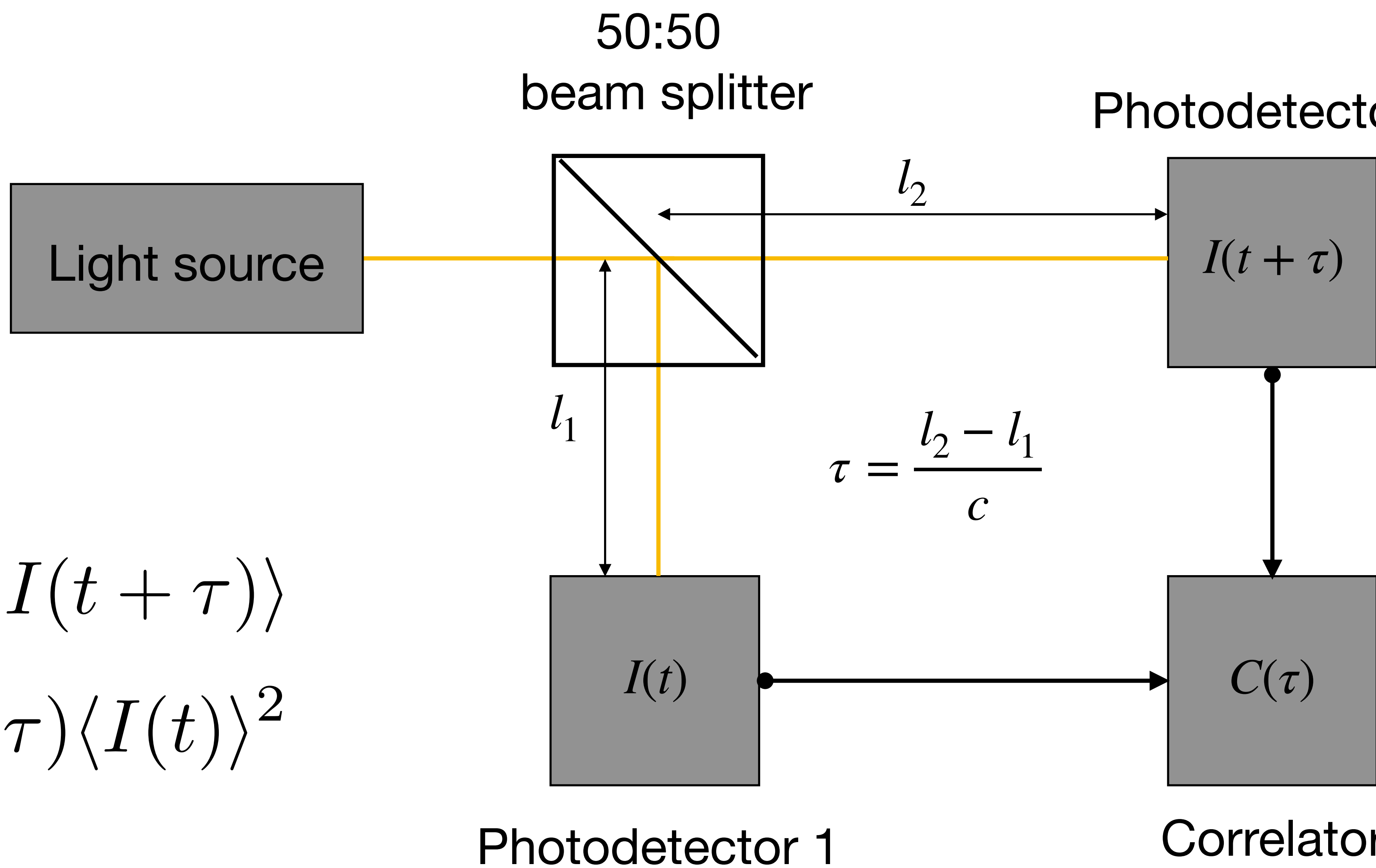
Hambury Brown-Twist experimental setup and correlation functions



$$C(\tau) = \langle I(t) I(t+\tau) \rangle$$
$$= g^{(2)}(\tau) \langle I(t) \rangle^2$$

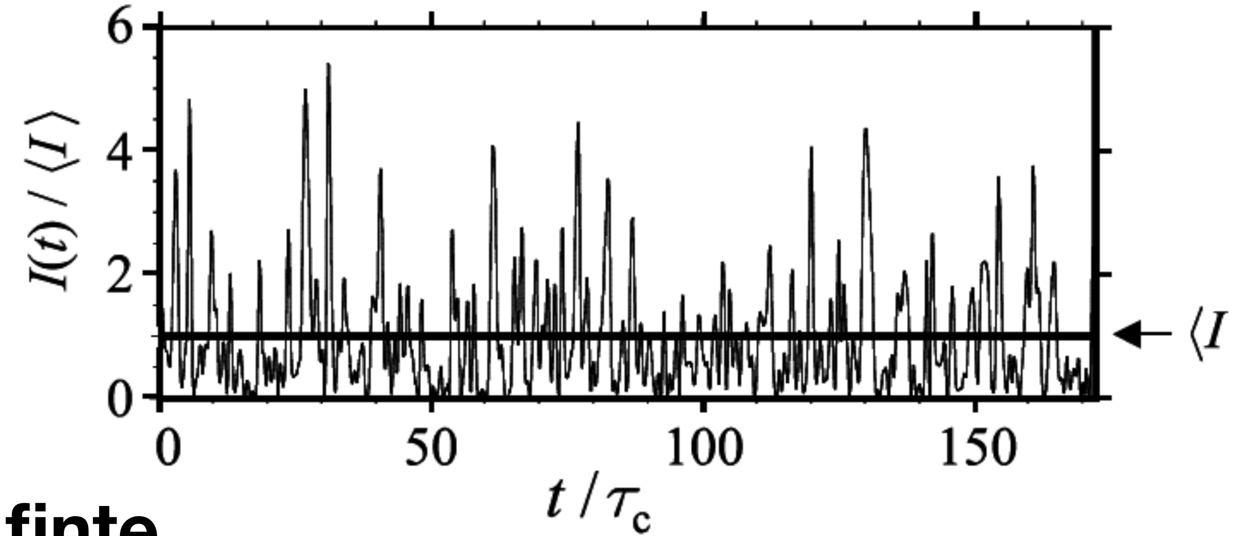
Second-order correlation function

$$g^{(2)}(\tau) = \frac{\langle I(t) I(t+\tau) \rangle}{\langle I(t) \rangle^2}$$

Second order correlation function for classical fields is bounded to be greater or equal to 1

Constant average intensity

$$I(t) = \langle I \rangle + \Delta I(t)$$



τ_c is finte

$$\tau \sim \tau_c$$

Correlated fluctuation

$$g^{(2)}(0) = \frac{\langle I(t)^2 \rangle}{\langle I(t) \rangle^2}$$

$$\tau \gg \tau_c$$

Uncorrelated fluctuation

$$g^{(2)}(\tau \gg \tau_c) = \frac{\langle I(t) I(t+\tau) \rangle}{\langle I(t) \rangle^2} = \frac{\langle I(t) \rangle^2}{\langle I(t) \rangle^2} = 1$$

τ_c does not exist

$$g^{(2)}(\tau) = 1$$

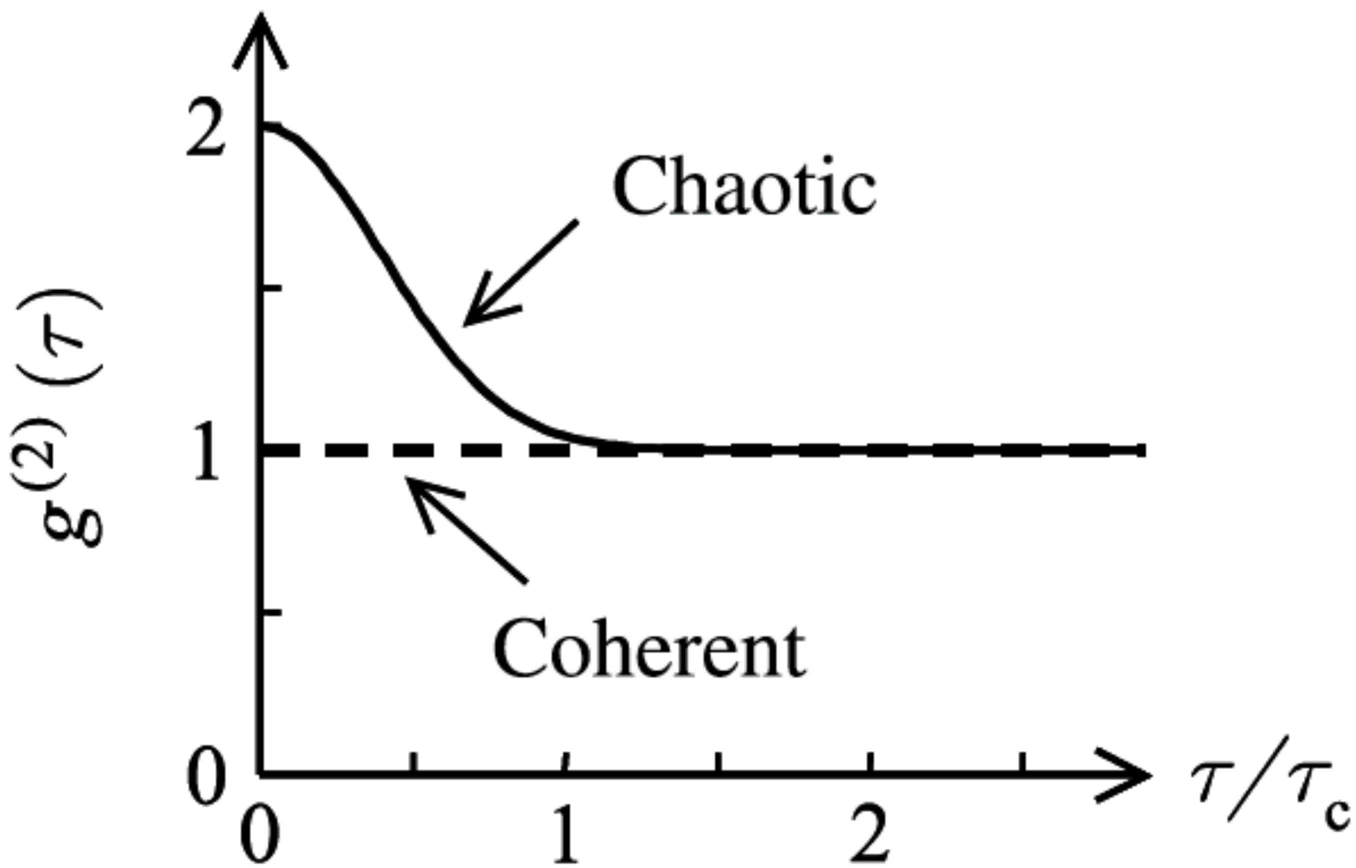
for any conceivable classical time dependance of *I(t)*

$$g^{(2)}(0) \geq 1$$

and

$$g^{(2)}(0) \geq g^{(2)}(\tau)$$

The form of the second-order correlation function for chaotic light can be calculated assuming simple model for the source



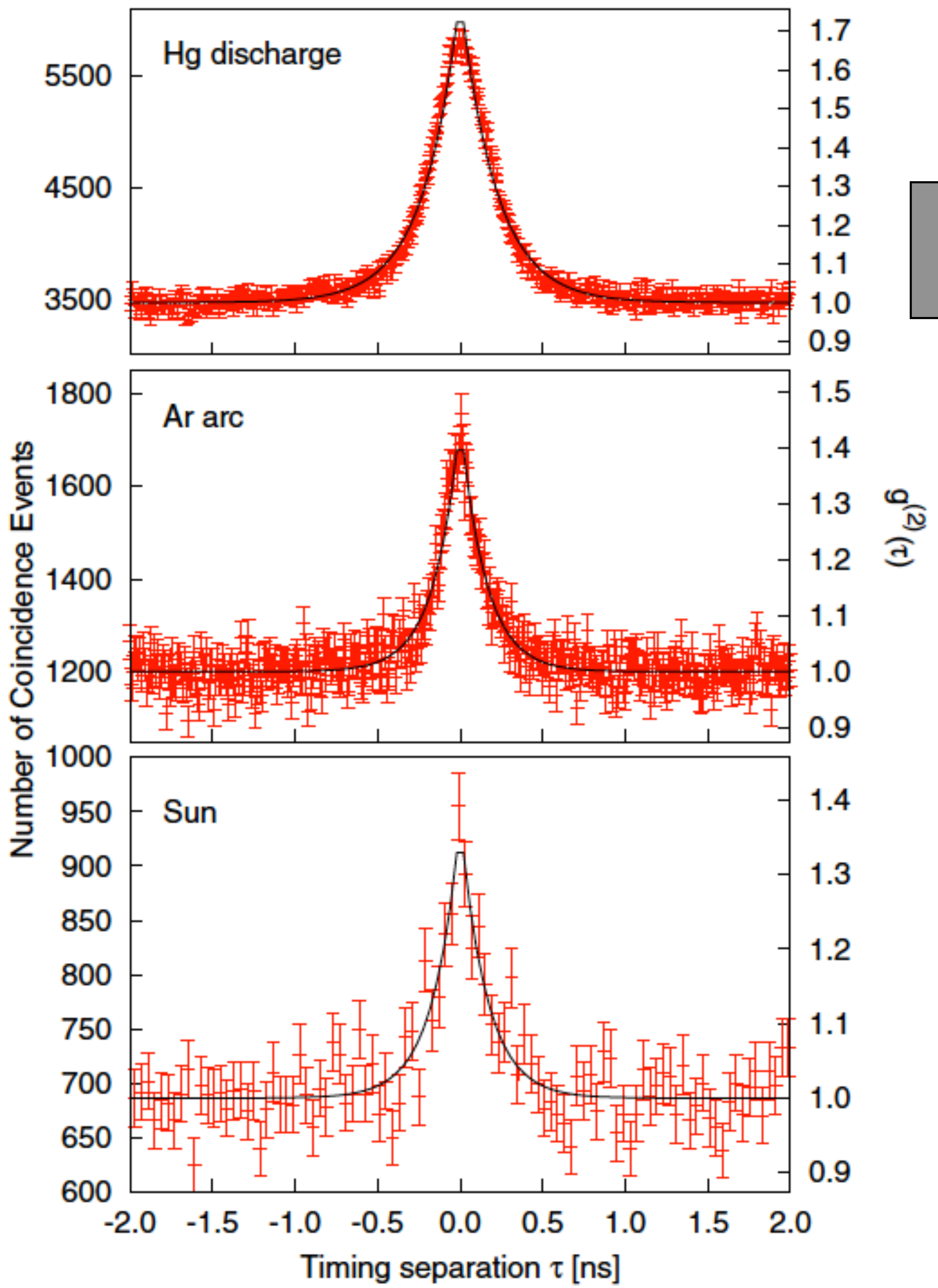
Lifetime broadening

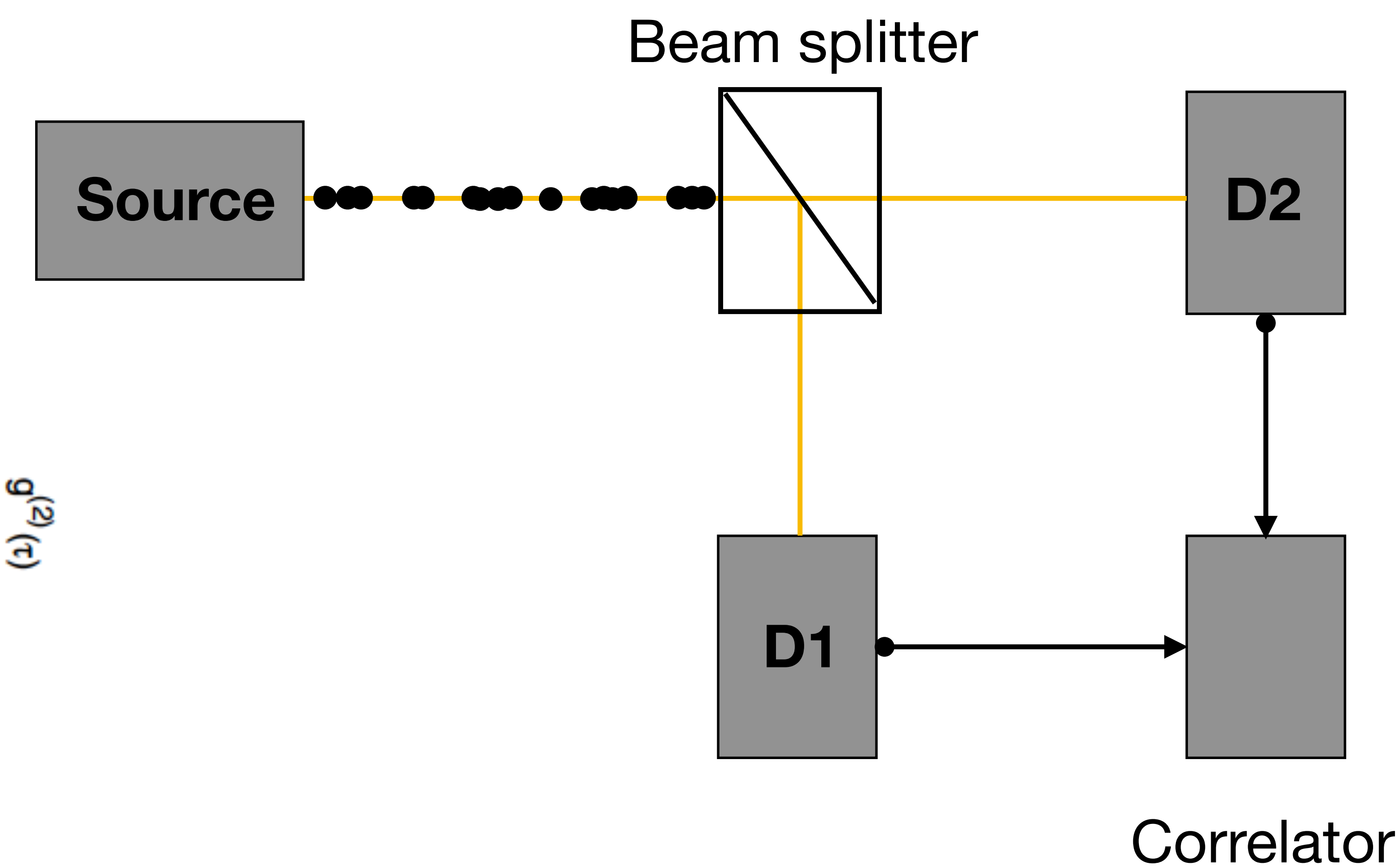
$$g^{(2)}(\tau) = 1 + \exp\left[-2\frac{\tau}{\tau_0}\right]$$

Doppler broadening

$$g^{(2)}(\tau) = 1 + \exp\left[-\pi\left(\frac{\tau}{\tau_c}\right)^2\right]$$

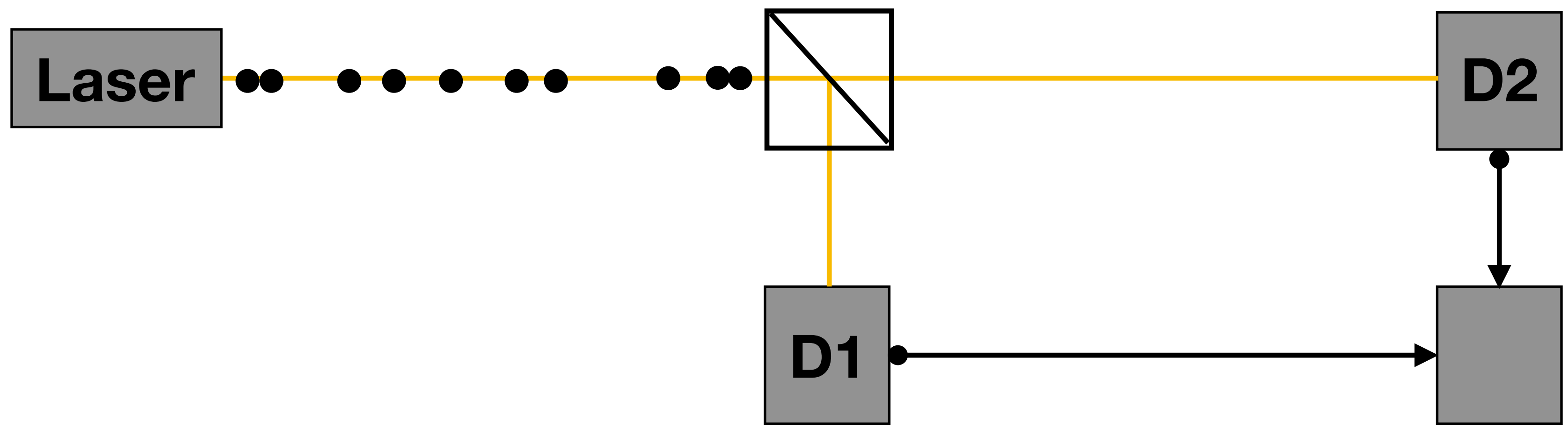
Thermal and chaotic light present the phenomenon of photon bunching





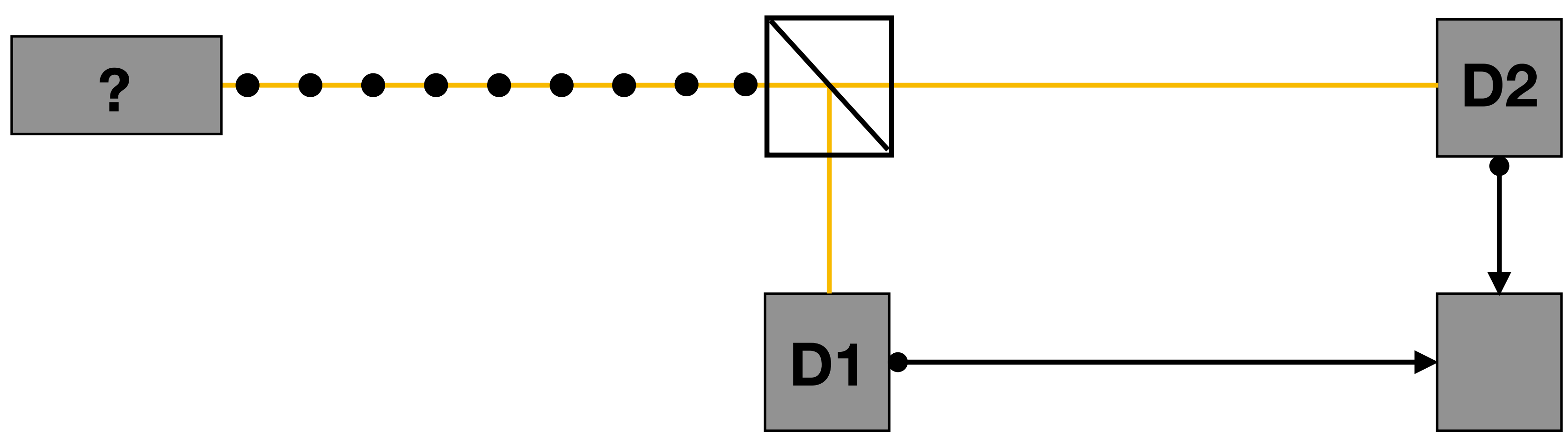
Photons tends to clump together in bunches

Anti-bunched light is a purely quantum effect



$$g^{(2)}(\tau) = 1 \qquad \forall\ \tau$$

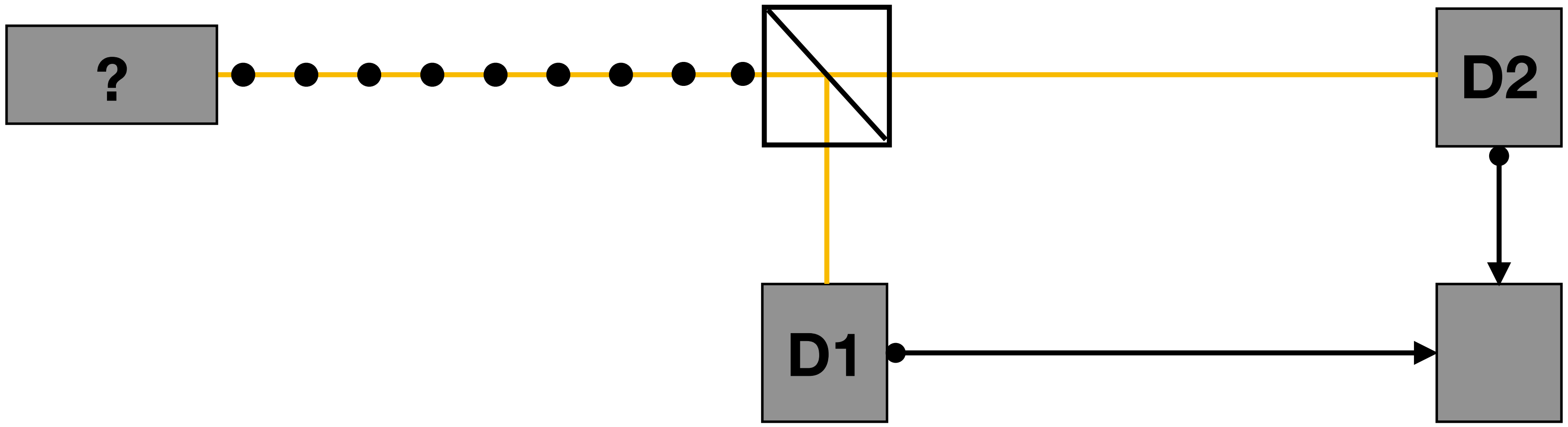
Photons are distributed randomly in the beam



Photons are distributed evenly with long time intervals between each others

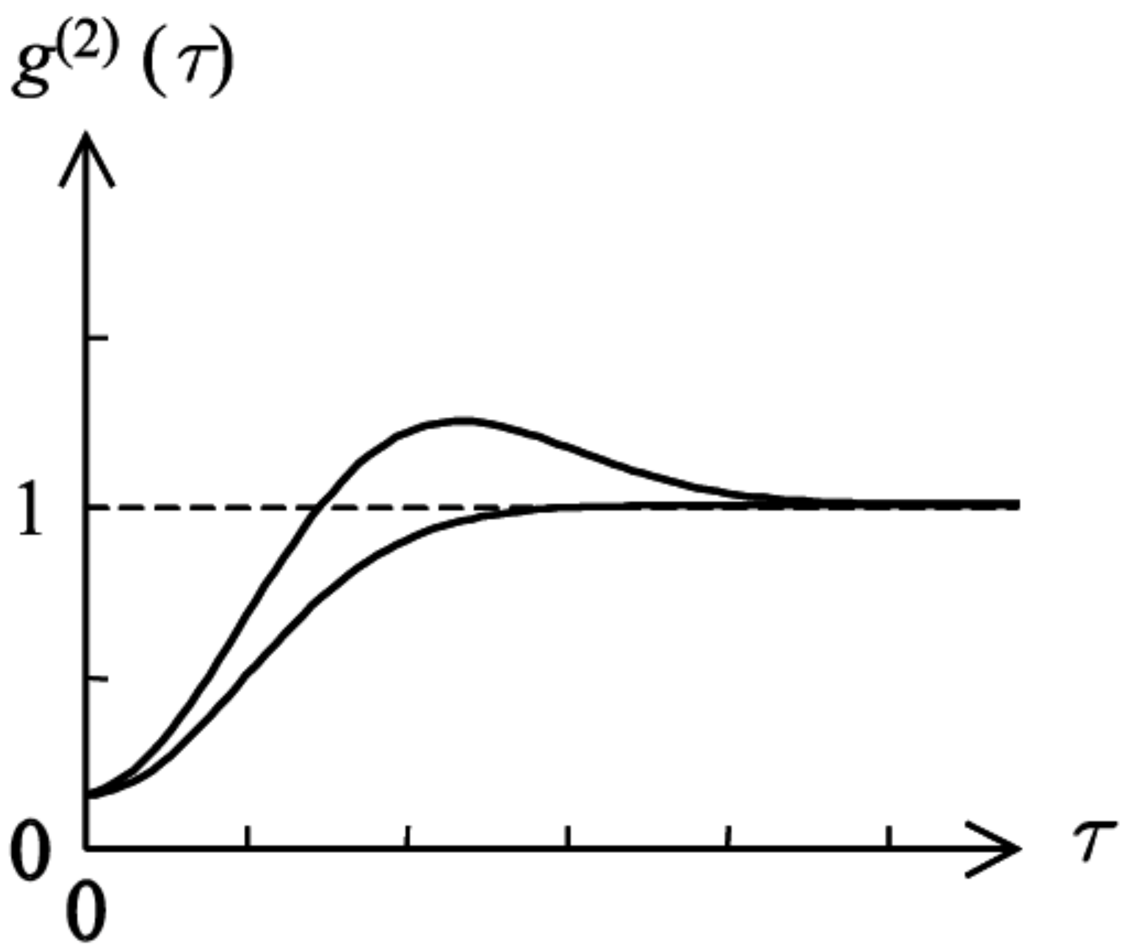
$$g^{(2)}(\tau) = ?$$

Anti-bunched light is a purely quantum effect



Photons are distributed evenly with long time intervals between each others

$$g^{(2)}(\tau) = ?$$

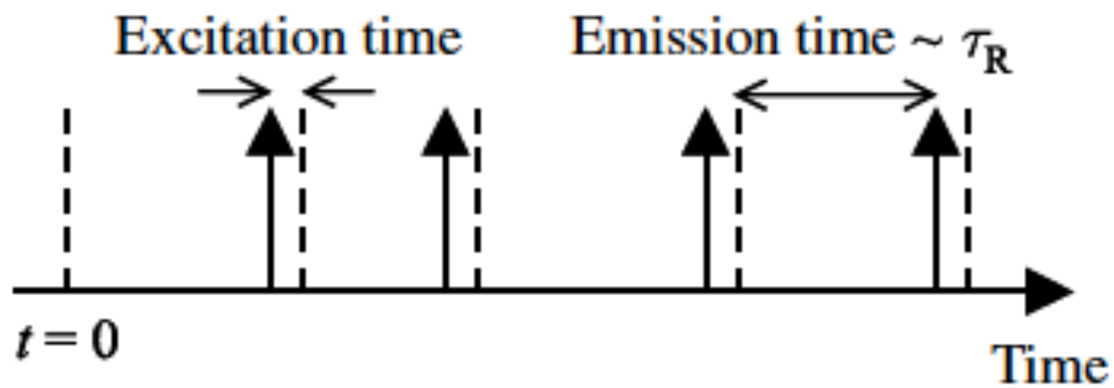


$$g^{(2)}(0) < g^{(2)}(\tau)$$

$$g^{(2)}(0) < 1$$

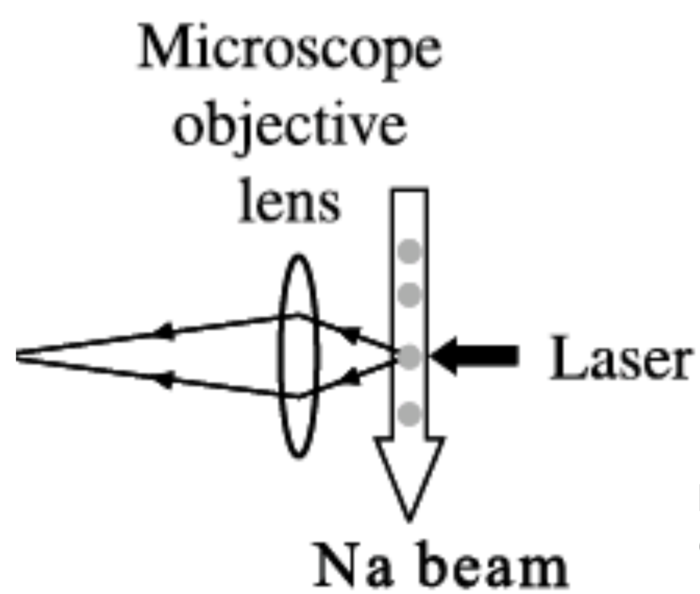
In contrast with the classical result

The basic idea for generating anti-bunched light is to isolate an individual emitting species



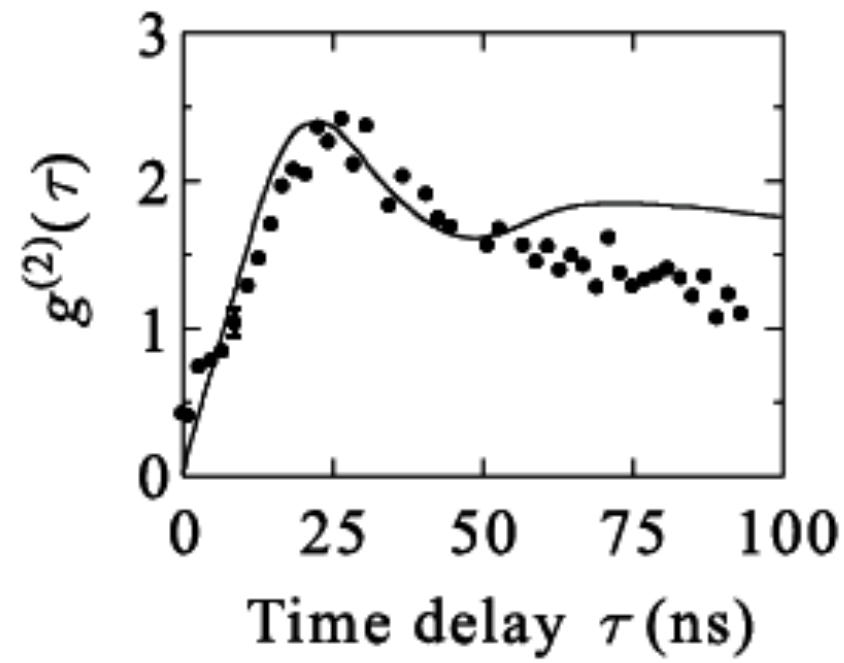
τ_R Radiative lifetime of the excited state

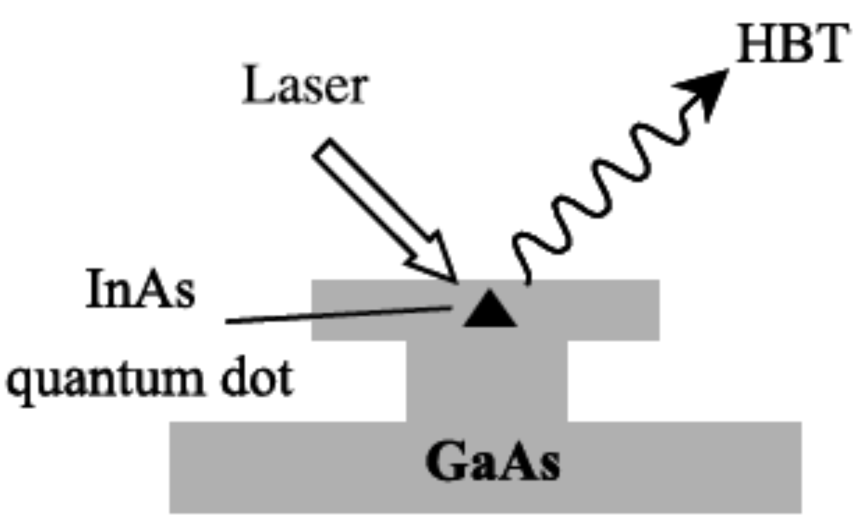
HBT



Resonant fluorescence of few atoms of Sodium

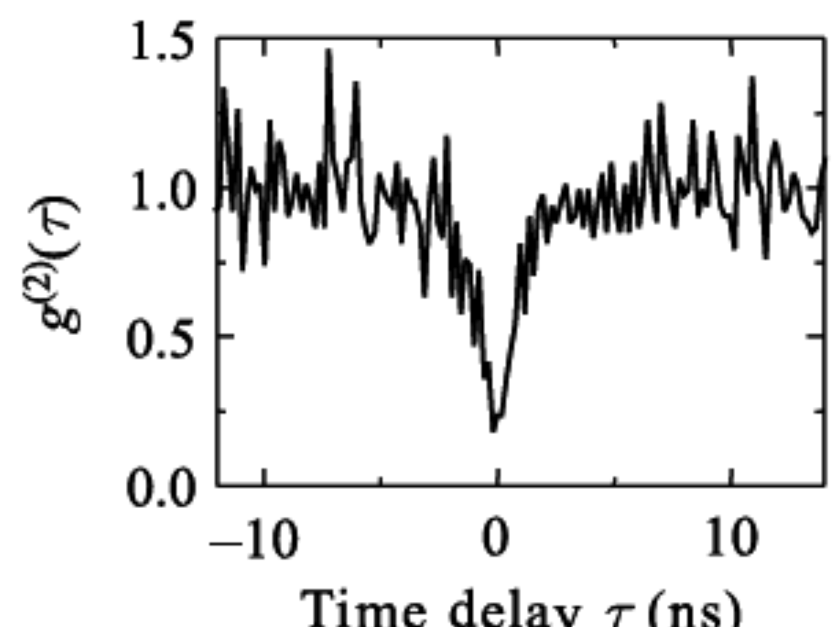
H.J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett., 39, 691 (1977)





Individual semiconductor quantum dot

P. Michler et al., Science 290, 2282 (2000)



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The value of $g^{(2)}(\tau)$ is crucial in determining the photon counting statistics

Considering the probability of detecting n photons in the time interval $[t, t+T]$

$$\langle(\Delta n)^2\rangle - \langle n\rangle = \langle n\rangle^2 \frac{1}{T^2}\int_{-T}^{T} d\tau (T - |\tau|)(g^{(2)}(\tau) - 1)$$

$g^{(2)}(\tau) > 1 \implies$ **super-Poissonian statistics**

$g^{(2)}(\tau) < 1 \implies$ **sub-Poissonian statistics**

not the other way around …

Anti-bunching and sub-Poissonian statistics are distinct phenomena

PHYSICAL REVIEW A VOLUME 41, NUMBER 1 1 JANUARY 1990

Photon-antibunching and sub-Poissonian photon statistics

X. T. Zou and L. Mandel
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 19 April 1989)

It is shown by example that sub-Poissonian photon-counting statistics need not imply photon antibunching, but can be accompanied by photon bunching, i.e., by the tendency of two photons to be close together more frequently than further apart. Some comments on the relation between antibunching and sub-Poissonian statistics are made.

One page paper:
a reference to an experiment ad a simple counter example to simply say “No”