A new formulation of Lee-Wick models and its implications for quantum gravity

Marco Piva

Pre-thesis seminar

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PhD course
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Outline

The problem of renormalizability in QG

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QED

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$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}(i\not{\partial} - e\not{A} - m)\psi, \quad \alpha = \frac{e^2}{4\pi}$$

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QED	Quantum Gravity
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In QG renormalization generates an infinite number of new counterterms.
A high-energy modification of the theory is necessary.

Higher-derivative quantum gravity

Stelle theory provides a renormalizable model of quantum gravity

$$S_{HD} = -\frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\big[R + \alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2\big].$$

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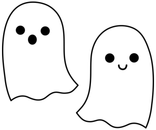

Unitarity

$$S^{\dagger}S = 1$$

$$-i(T - T^{\dagger}) = T^{\dagger}T, \qquad S = 1 + iT;$$

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- Higher derivatives $\Rightarrow$ improve the convergence;
- Complex conjugated poles $\Rightarrow$ trivial imaginary part.

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I) Nonlocal, non-Hermitian divergences (also in 4-dim. QG).

$$\Sigma(p) = -\frac{M^4}{2(4\pi)^3}\left[\frac{M^2}{(p^2)^2} - \frac{i}{p^2}\right]\ln\left(\frac{\Lambda_{UV}}{M^2}\right) + \ldots, \qquad D = 6.$$

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The theory cannot be defined directly on Minkowski spacetime.

Lee-Wick quantum field theories

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Our formulation solves all these problems.

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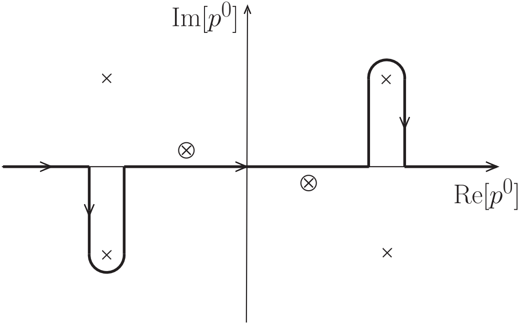

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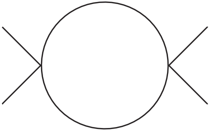

$$\mathcal{J}(p) = \int \frac{\mathrm{d}^D k}{(2\pi)^D} D(k^2, m_1^2, \epsilon_1) D((k-p)^2, m_2^2, \epsilon_2) \quad (1)$$

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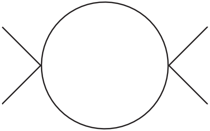

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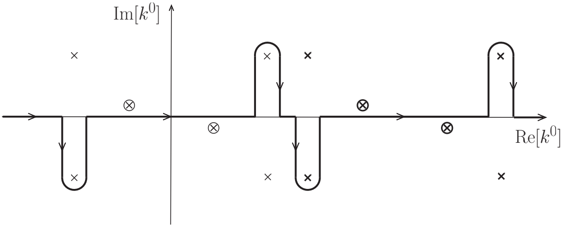

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Solution: define LW models as
Nonanalytically Wick rotated Euclidean theories.

D. Anselmi and M. Piva, *A new formulation of Lee-Wick quantum field theory*, JHEP 1706 (2017) 066, 17A1 Renormalization.com and arXiv:1703.04584 [hep-th].

New formulation

Analytic structure of $\mathcal{J}(p)$ in the p^0 complex plane

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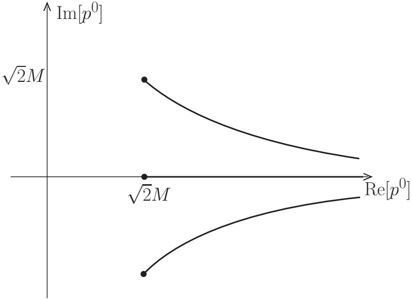

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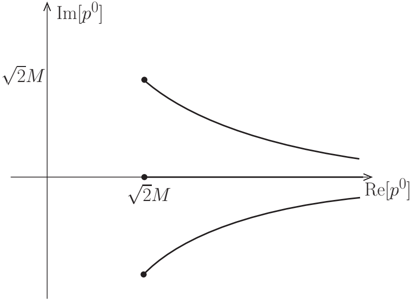

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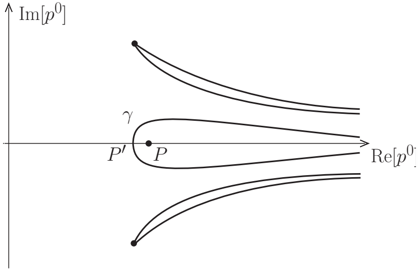
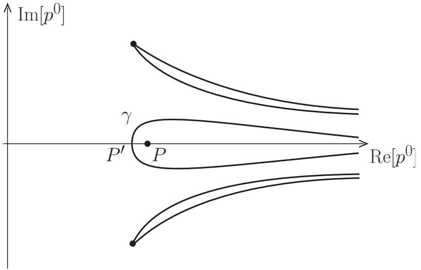
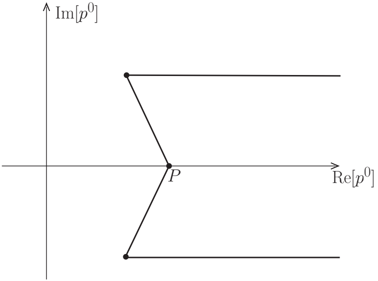
$\mathbf{p} \neq 0$



Lorentz invariance seems violated (already noticed by Nakanishi)

Deform the branch cuts $\leftrightarrow$ Deform the $\mathbf{k}$ integration domain





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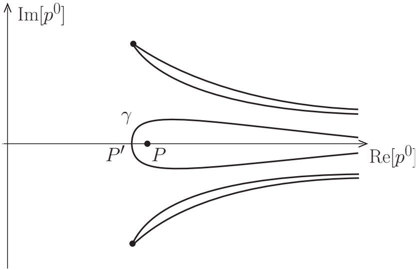

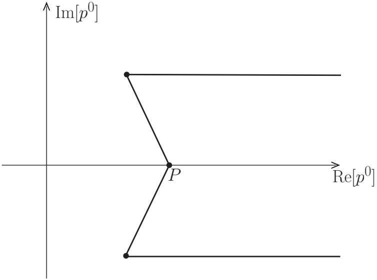

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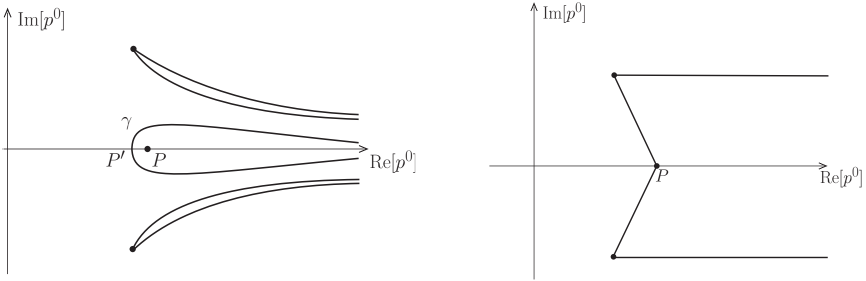

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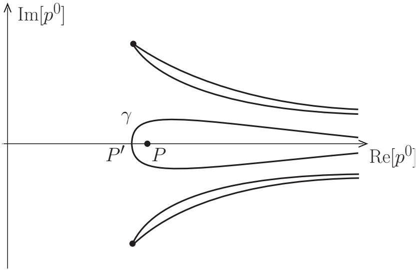

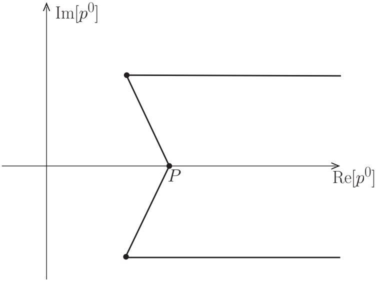

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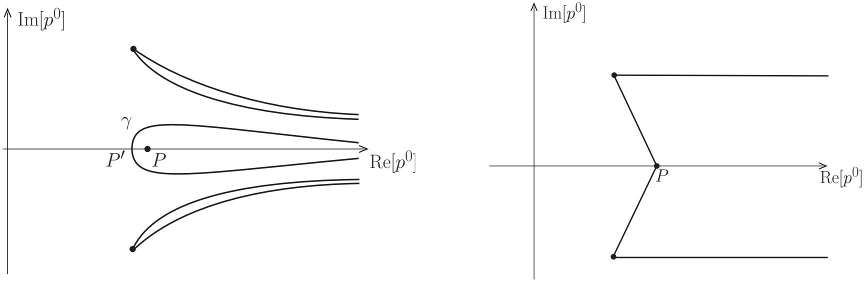

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The amplitude are well defined but nonanalytic.

The deformation is practically hard to implement

We argue that

$$\mathcal{J}^{>}_{\rm LW}(p) = \frac{1}{2}\left[\mathcal{J}^{0+}_{\rm LW}(p) + \mathcal{J}^{0-}_{\rm LW}(p)\right]$$

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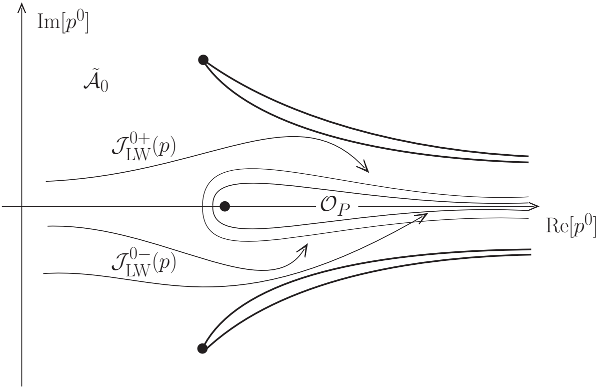

Expansion around the pinching (τ, η fluctuations)

$$D_\varphi = p^0 \mathrm{e}^{i\varphi} - \omega_M(\mathbf{k}) - \omega_M^*(\mathbf{k}-\mathbf{p}), \quad \omega_M(\mathbf{k}) = \sqrt{\mathbf{k}^2 + iM^2}, \quad p_s = |\mathbf{p}|.$$

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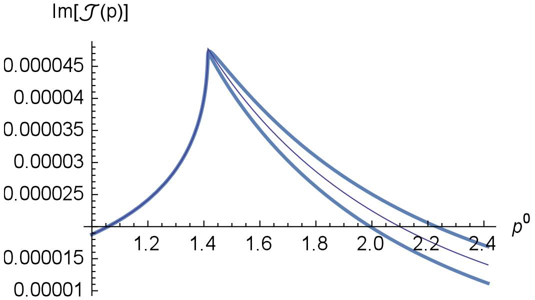
No prescription with nonvanishing a is consistent with this formulation.

CLOP gives $a = \pi \mathrm{sgn}(M' - M)$

Discrepancy above the threshold

$$M = 1,\ \delta = \pm 10^{-3},\ m_1 = 3,\ m_2 = 5,\ \delta = -i(M - M').$$



Our formulation gives physical predictions which differ from the previous ones.

Lee-Wick quantum gravity

$$\mathcal{L}_{\mathrm{QG}} = -\frac{1}{2\kappa^2}\sqrt{-g}\Big[R - \frac{1}{M^4}\big(D_\rho R_{\mu\nu}\big)\big(D^\rho R^{\mu\nu}\big) + \frac{1}{2M^4}\big(D_\rho R\big)\big(D^\rho R\big)\Big]$$

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Expansion $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}, \qquad \eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1).$

Propagator in harmonic gauge

$$\langle h_{\mu\nu}(p) h_{\rho\sigma}(-p)\rangle^{\mathrm{free}} = \frac{iM^4}{2(p^2 + i\epsilon)} \frac{\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}}{(p^2)^2 + M^4}.$$

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Problem of uniquness

Superrenormalizable models of quantum gravity are infinitely many.

New quantization prescription

D. Anselmi, *On the quantum field theory of gravitational interactions*, JHEP 1706 (2017) 086, 17A3 Renormalization.com and arXiv:1704.07728 [hep-th].

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$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\lambda}{4!}\varphi^4$$

Modified Euclidean propagator

$$\frac{p_E^2}{(p_E^2)^2 + \mathcal{E}^4}, \quad \mathcal{E} = \text{ ficticious LW scale}$$

New prescription

$$\lim_{\mathcal{E}\to 0} \frac{p^2}{[(p^2)^2 + \mathcal{E}^4]_{\text{LW}}}$$

Quantum gravity with dimensionless gauge couplings

$$\mathcal{L}_{QG} = -\frac{1}{2\kappa^2}\sqrt{-g}\left[2\Lambda_C + \zeta R - \frac{\gamma}{M^2}R_{\mu\nu}R^{\mu\nu} + \frac{1}{2M^2}(\gamma - \eta)R^2\right], \quad \zeta > 0,\ \gamma < 0.$$

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A unique, renormalizable and unitary theory of QG in 4 dim.

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A unique, renormalizable and unitary theory of QG in 4 dim.

Unitarity can be proved only if $\Lambda_C = 0$ but realistic models have $\Lambda_C \neq 0$.
The cosmological constant might be an anomaly of unitarity.

Conclusions and future developments

- Solution of the quantum gravity problem: reconcile renormalizability and unitarity.

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