Particle physics and anomalies

Marco Piva

PhD course
Università di Pisa

June 10, 2016

The success of quantum field theory

- Quantum theory + special relativity.

The success of quantum field theory

- Quantum theory + special relativity.
- Scattering processes and decays for elementary particles.

The success of quantum field theory

- Quantum theory + special relativity.
- Scattering processes and decays for elementary particles.
- Description of non elementary particles (mesons, baryons).

The success of quantum field theory

- Quantum theory + special relativity.
- Scattering processes and decays for elementary particles.
- Description of non elementary particles (mesons, baryons).
- Energy dependence of the parameters (renormalization).

The success of quantum field theory

- Quantum theory + special relativity.
- Scattering processes and decays for elementary particles.
- Description of non elementary particles (mesons, baryons).
- Energy dependence of the parameters (renormalization).
- Explain how particles get mass (Higgs mechanism).

The success of quantum field theory

- Quantum theory + special relativity.
- Scattering processes and decays for elementary particles.
- Description of non elementary particles (mesons, baryons).
- Energy dependence of the parameters (renormalization).
- Explain how particles get mass (Higgs mechanism).
- Theory of 3/4 fundamental interactions (standard model).

Outline

Classical field theory

Lagrangian density $\mathcal{L}(\phi, \partial\phi)$;
continous tranformation $\mathcal{T} : \phi \to \phi'(\phi, \alpha)$.

Classical field theory

Lagrangian density $\mathcal{L}(\phi, \partial\phi)$;
continous tranformation $\mathcal{T} : \phi \to \phi'(\phi, \alpha)$.

If $\mathcal{T} : \mathcal{L} \to \mathcal{L}' = \mathcal{L} + \partial_\mu \mathcal{J}^\mu$, then $\mathcal{T}$ is called a **symmetry** of $\mathcal{L}$.

Classical field theory

Lagrangian density $\mathcal{L}(\phi, \partial\phi)$;
continous tranformation $\mathcal{T} : \phi \to \phi'(\phi, \alpha)$.

If $\mathcal{T} : \mathcal{L} \to \mathcal{L}' = \mathcal{L} + \partial_\mu \mathcal{J}^\mu$, then $\mathcal{T}$ is called a **symmetry** of $\mathcal{L}$.

Global symmetry: α does not depend on spacetime points.

Classical field theory

Lagrangian density $\mathcal{L}(\phi, \partial\phi)$;
continous tranformation $\mathcal{T} : \phi \to \phi'(\phi, \alpha)$.

If $\mathcal{T} : \mathcal{L} \to \mathcal{L}' = \mathcal{L} + \partial_\mu \mathcal{J}^\mu$, then $\mathcal{T}$ is called a **symmetry** of $\mathcal{L}$.

Global symmetry: α does not depend on spacetime points.
Gauge symmetry: α depends on spacetime points.

Classical field theory

Lagrangian density $\mathcal{L}(\phi, \partial\phi)$;
continous tranformation $\mathcal{T} : \phi \to \phi'(\phi, \alpha)$.

If $\mathcal{T} : \mathcal{L} \to \mathcal{L}' = \mathcal{L} + \partial_\mu \mathcal{J}^\mu$, then $\mathcal{T}$ is called a **symmetry** of $\mathcal{L}$.

Global symmetry: α does not depend on spacetime points.
Gauge symmetry: α depends on spacetime points.

Noether theorem

$\forall$ symmetry of $\mathcal{L}$, $\exists$ a conserved density current J^μ

$$\partial_\mu J^\mu = 0. \tag{1}$$

Quantum field theory

Starting from a classical Lagrangian density $\mathcal{L}$.

$\downarrow$

Perturbation theory (Feynman diagrams).

Quantum field theory

Starting from a classical Lagrangian density $\mathcal{L}$.

$\downarrow$

Perturbation theory (Feynman diagrams).

$\downarrow$

Quantum corrections.

Quantum field theory

Starting from a classical Lagrangian density $\mathcal{L}$.

$\downarrow$

Perturbation theory (Feynman diagrams).

$\downarrow$

Quantum corrections.

Operator $\mathcal{O}(\phi, \partial\phi) \rightarrow$ expectation value $\langle \mathcal{O}(\phi, \partial\phi) \rangle$.

Anomalies

Quantum corrections can give non zero contributions to the operator $\partial_\mu J^\mu$ such that

$$\langle \partial_\mu J^\mu \rangle \neq 0. \tag{2}$$

Anomalies

Quantum corrections can give non zero contributions to the operator $\partial_\mu J^\mu$ such that

$$\langle \partial_\mu J^\mu \rangle \neq 0. \tag{2}$$

We call $\mathcal{A} = \langle \partial_\mu J^\mu \rangle$ **anomaly**.

The π^0 decay problem

$\boldsymbol{\pi^0}$: neutral, $P = -1$, spin-0 meson.

The π^0 decay problem

$\boldsymbol{\pi^0}$: neutral, $P = -1$, spin-0 meson. **Meson**: $q - \bar{q}$ bound state.

The π^0 decay problem

$\boldsymbol{\pi^0}$: neutral, $P = -1$, spin-0 meson. **Meson**: $q - \bar{q}$ bound state.

Consider the electromagnetic decay

$$\pi^0 \to \gamma\gamma. \tag{3}$$

The π^0 decay problem

$\boldsymbol{\pi^0}$: neutral, $P = -1$, spin-0 meson. **Meson**: $q - \bar{q}$ bound state.

Consider the electromagnetic decay

$$\pi^0 \to \gamma\gamma. \tag{3}$$

In 1967, Sutherland and Veltman shown that:
exact symmetries at quantum level $\Rightarrow \left\langle \gamma\gamma | \pi^0 \right\rangle \to 0$ for $m_\pi \to 0$.

The π^0 decay problem

$\boldsymbol{\pi^0}$: neutral, $P=-1$, spin-0 meson. **Meson**: $q-\bar{q}$ bound state.

Consider the electromagnetic decay

$$\pi^0 \to \gamma\gamma. \tag{3}$$

In 1967, Sutherland and Veltman shown that:
exact symmetries at quantum level $\Rightarrow \left\langle \gamma\gamma|\pi^0 \right\rangle \to 0$ for $m_\pi \to 0$.
The only operator is $C\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$, symmetry $\Rightarrow C=0$.

The π^0 decay problem

$\boldsymbol{\pi^0}$: neutral, $P = -1$, spin-0 meson. **Meson**: $q - \bar{q}$ bound state.

Consider the electromagnetic decay

$$\pi^0 \to \gamma\gamma. \tag{3}$$

In 1967, Sutherland and Veltman shown that:
exact symmetries at quantum level $\Rightarrow \left\langle \gamma\gamma | \pi^0 \right\rangle \to 0$ for $m_\pi \to 0$.
The only operator is $C \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$, symmetry $\Rightarrow C = 0$.

Observed decay rate $\Gamma_{\text{obs}}^{\pi\gamma\gamma} = (1,19 \pm 0,08) \times 10^{16} s^{-1}$
(Corresponding to $\sim 98\%$ of the decays).

The π^0 decay problem

$\boldsymbol{\pi^0}$: neutral, $P = -1$, spin-0 meson. **Meson**: $q - \bar{q}$ bound state.

Consider the electromagnetic decay

$$\pi^0 \to \gamma\gamma. \tag{3}$$

In 1967, Sutherland and Veltman shown that:
exact symmetries at quantum level $\Rightarrow \langle \gamma\gamma | \pi^0 \rangle \to 0$ for $m_\pi \to 0$.
The only operator is $C\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$, symmetry $\Rightarrow C = 0$.

Observed decay rate $\Gamma^{\pi\gamma\gamma}_{\text{obs}} = (1,19 \pm 0,08) \times 10^{16} s^{-1}$
(Corresponding to $\sim 98\%$ of the decays).

Solution: **axial anomaly**.

Axial anomaly

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{\partial}\psi - e\bar{\psi}\not{A}\psi,$$

Axial anomaly

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{\partial}\psi - e\bar{\psi}\not{A}\psi,$$

invariant under a global $U(1)$ symmetry

$$\psi \to e^{i\alpha\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}.$$

Axial anomaly

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\partial\!\!\!/\psi - e\bar{\psi}A\!\!\!/\psi,$$

invariant under a global $U(1)$ symmetry

$$\psi \to e^{i\alpha\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}.$$

Adler-Bell-Jackiw

Classical level: $\partial_\mu J_5^\mu = 0.$ Quantum level: $\langle \partial_\mu J_5^\mu \rangle \neq 0.$

Axial anomaly

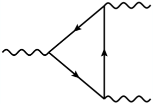
$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\partial\!\!\!/\psi - e\bar{\psi}A\!\!\!/\psi,$$

invariant under a global $U(1)$ symmetry

$$\psi \to e^{i\alpha\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}.$$

Adler-Bell-Jackiw
Classical level: $\partial_\mu J_5^\mu = 0$. Quantum level: $\langle \partial_\mu J_5^\mu \rangle \neq 0$.

Quantum correction due to the diagram



ABJ anomaly, N_c colors: $\mathcal{A} = \langle \partial_\mu J_5^\mu \rangle_{\text{one-loop}} = \frac{N_c e^2}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$.

ABJ anomaly, N_c colors: $\mathcal{A} = \langle \partial_\mu J_5^\mu \rangle_{\text{one-loop}} = \frac{N_c e^2}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$.

$\mathcal{A}$ has a physical meaning:
$\partial_\mu J_5^\mu$ composite operator with the same quantum numbers of π^0.

$\Downarrow$

Processes involving π^0 can be described by correlation functions involving $\partial_\mu J_5^\mu$.

ABJ anomaly, N_c colors: $\mathcal{A} = \langle \partial_\mu J_5^\mu \rangle_{\text{one-loop}} = \frac{N_c e^2}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$.

$\mathcal{A}$ has a physical meaning:
$\partial_\mu J_5^\mu$ composite operator with the same quantum numbers of π^0.

$\Downarrow$

Processes involving π^0 can be described by correlation functions involving $\partial_\mu J_5^\mu$.

$$\Gamma_{ABJ}^{\pi\gamma\gamma} = \left(\frac{N_c}{3}\right)^2 \times 1,11 \times 10^{16} s^{-1}, \quad (4)$$

$$\Gamma_{\text{obs}}^{\pi\gamma\gamma} = (1,19 \pm 0,08) \times 10^{16} s^{-1}. \quad (5)$$

$N_c = 3$ gives a result in agreement with the experiments.

Gauge anomalies

Global symmetries can be anomalous, leading to physical consequences.

Gauge anomalies

Global symmetries can be anomalous, leading to physical consequences.
Gauge symmetries are used to remove unphysical d.o.f.

If a gauge symmetry were anomalous, then the theory would be **non-unitary**.

Gauge anomalies

Global symmetries can be anomalous, leading to physical consequences.
Gauge symmetries are used to remove unphysical d.o.f.

If a gauge symmetry were anomalous, then the theory would be **non-unitary**.

We have a consistency condition for gauge theories:
All gauge anomalies must cancel!

Adler-Bardeen theorem

Higher order?

$$\langle \partial_\mu J_5^\mu \rangle = \frac{N_c e^2}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \dots$$

Adler-Bardeen theorem

Higher order?

$$\langle \partial_\mu J_5^\mu \rangle = \frac{N_c e^2}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \dots$$

Theorem

I) The ABJ axial anomaly is one-loop exact (Adler-Bardeen, 1969);

Adler-Bardeen theorem

Higher order?

$$\langle \partial_\mu J_5^\mu \rangle = \frac{N_c e^2}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \dots$$

Theorem

I) The ABJ axial anomaly is one-loop exact (Adler-Bardeen, 1969);

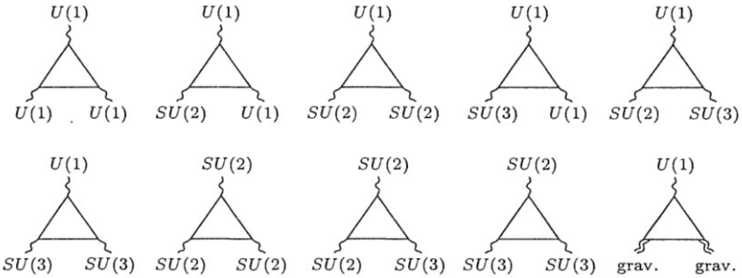
II) If gauge anomalies vanish at one-loop, they vanish to all orders.

Anomaly cancellation in the standard model

Standard model gauge group $G = SU(3)_c \times SU(2)_L \times U(1)_Y$.

Anomaly cancellation in the standard model

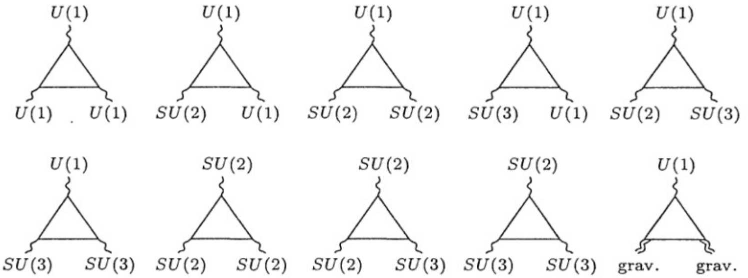
Standard model gauge group $G = SU(3)_c \times SU(2)_L \times U(1)_Y$.



Cancellations between quark and leptons are necessary.

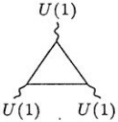
Anomaly cancellation in the standard model

Standard model gauge group $G = SU(3)_c \times SU(2)_L \times U(1)_Y$.

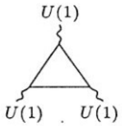


Cancellations between quark and leptons are necessary.

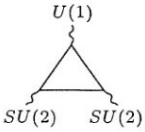
All one-loop anomalies cancel:
the standard model is anomaly free!



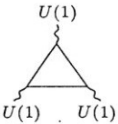
$$3 \cdot 2 \cdot \left(\frac{1}{3}\right)^3 - 3 \cdot \left(\frac{4}{3}\right)^3 - 3 \cdot \left(-\frac{2}{3}\right)^3 + 2 \cdot (-1)^3 - 1 \cdot (-2)^3 = 0$$



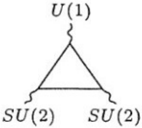
$$3 \cdot 2 \cdot \left(\frac{1}{3}\right)^3 - 3 \cdot \left(\frac{4}{3}\right)^3 - 3 \cdot \left(-\frac{2}{3}\right)^3 + 2 \cdot (-1)^3 - 1 \cdot (-2)^3 = 0$$



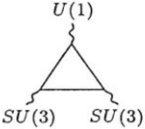
$$3 \cdot 2 \cdot \frac{1}{3} + 1 \cdot 2 \cdot (-1) = 0$$



$$3 \cdot 2 \cdot \left(\frac{1}{3}\right)^3 - 3 \cdot \left(\frac{4}{3}\right)^3 - 3 \cdot \left(-\frac{2}{3}\right)^3 + 2 \cdot (-1)^3 - 1 \cdot (-2)^3 = 0$$



$$3 \cdot 2 \cdot \frac{1}{3} + 1 \cdot 2 \cdot (-1) = 0$$



$$3 \cdot 2 \cdot \frac{1}{3} - 3 \cdot \frac{4}{3} - 3\left(-\frac{2}{3}\right) = 0$$

Conclusions

Anomalies are an important tool in quantum field theory.

- Solve the theoretical problem of π^0 decay.

Conclusions

Anomalies are an important tool in quantum field theory.

- Solve the theoretical problem of π^0 decay.
- Provide a check for the consistency of a theory .

Conclusions

Anomalies are an important tool in quantum field theory.

- Solve the theoretical problem of π^0 decay.
- Provide a check for the consistency of a theory .
- Predictivity: constraints on particle content in the standard model.