PROBING ASTROPHYSICAL BLACK HOLES WITH RINGDOWN SIGNALS

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UNIVERSITÀ DI PISA

PISA, 23 September '19
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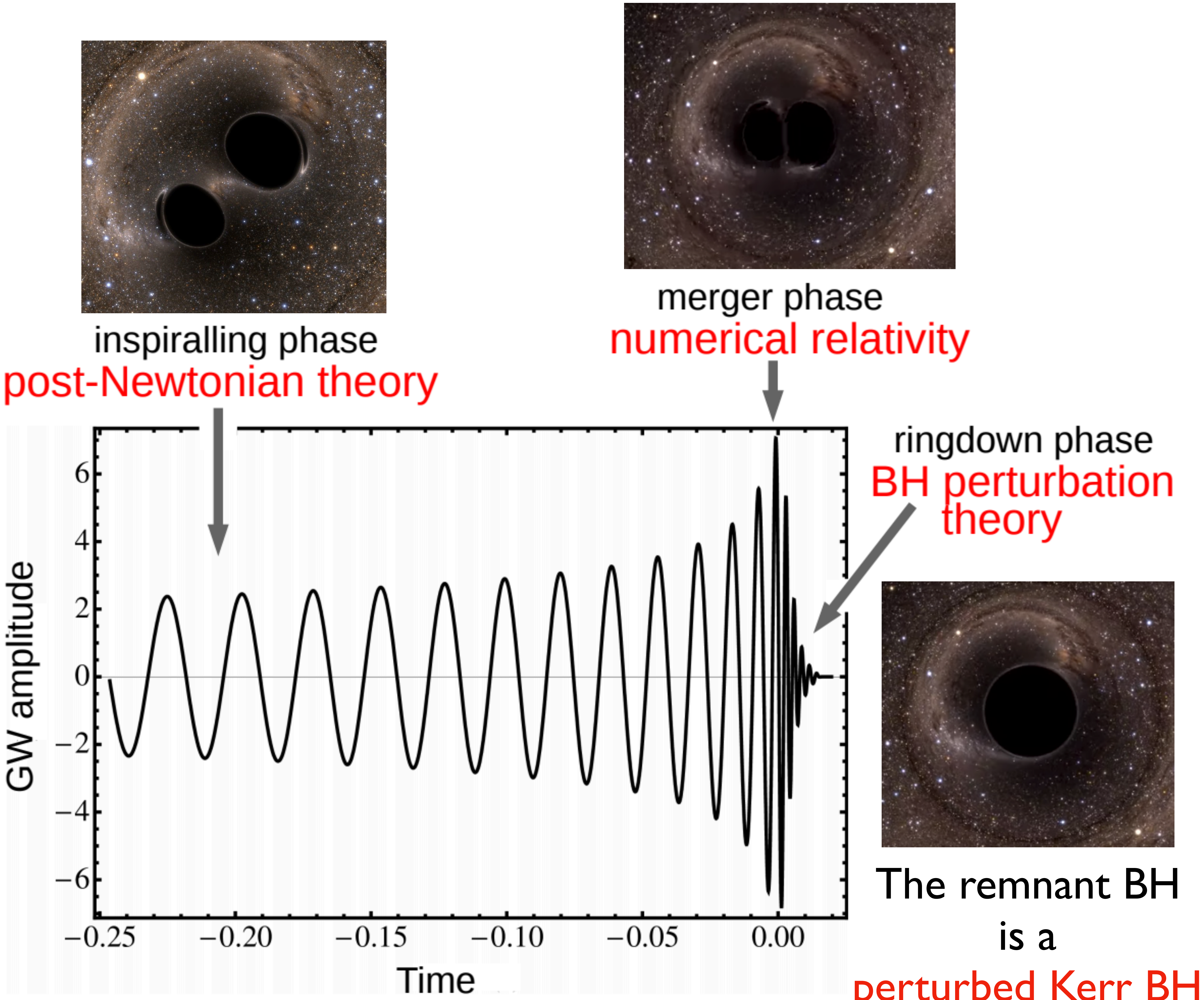
23 September 2019

CONTENTS

INTRODUCTION (I): THEORY



The remnant BH is a perturbed Kerr BH

[Sources: Blanchet, arXiv:1902.09801;
SXS Project: http://www.black-holes.org]

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- Perturbation theory is governed by Teukolsky's equation (TE)
- TE is a PDE separable in radial and angular ODEs
- TE solution can be expressed through the complex strain:

$$h_+ - i h_\times = \frac{M_f}{D_L} \sum_{lmn} \left\{ \tilde{A}_{lmn} \, {}_{-2}S_{lmn}(a_f \, \tilde{\omega}_{lmn}, \iota, \Phi) e^{i(t - t_{lmn})\tilde{\omega}_{lmn}} + \text{c.c.} \right\}$$

where:

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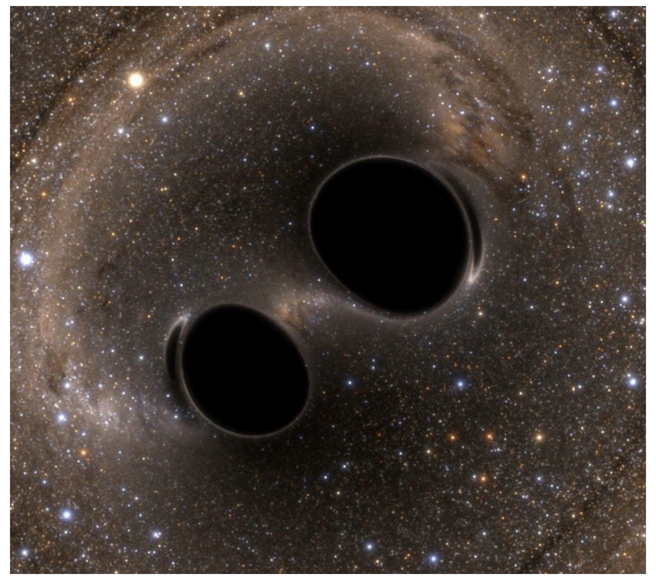
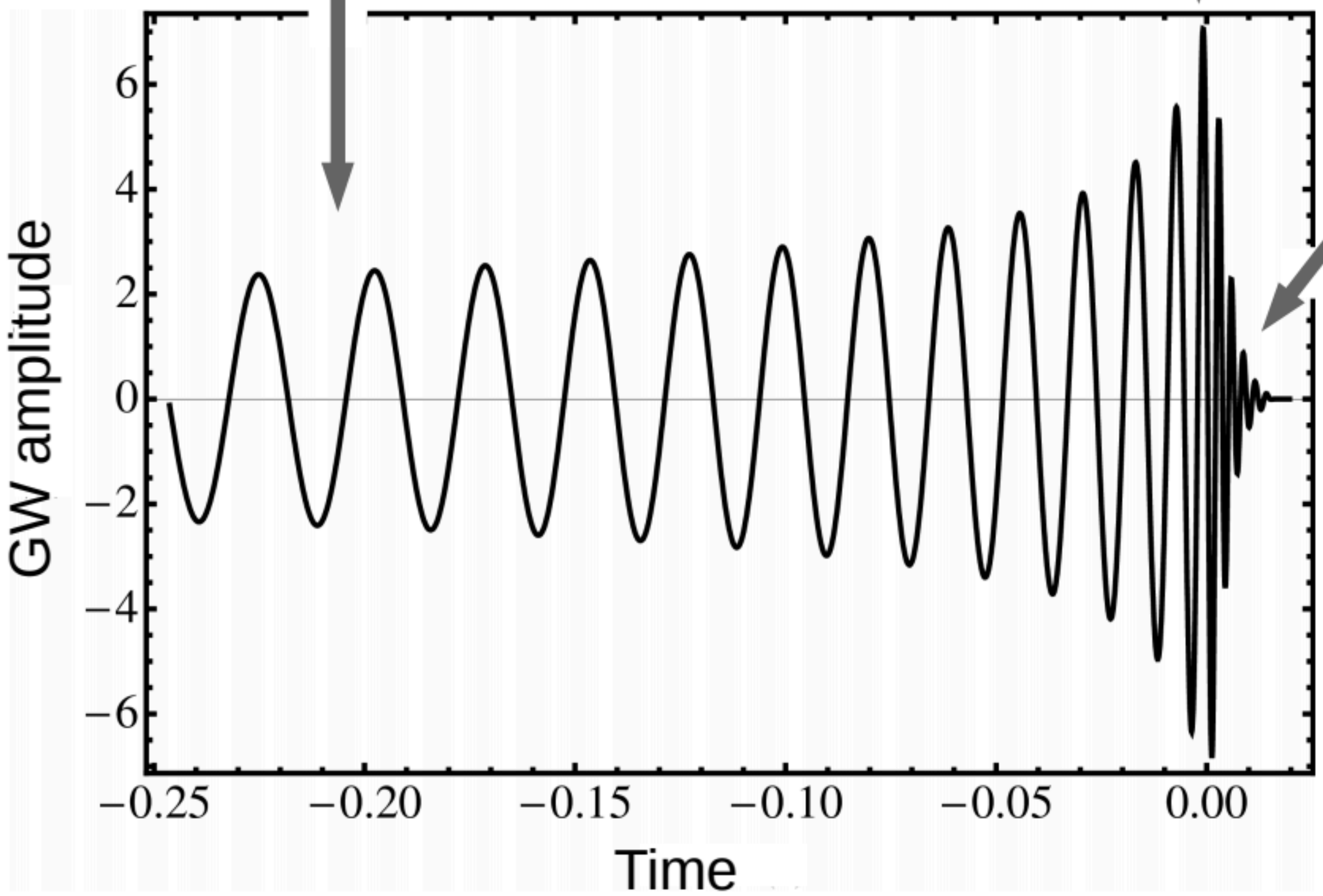
$$\tilde{\omega}_{lmn}(M_f, a_f) = \omega_{lmn}(M_f, a_f) + i/\tau_{lmn}(M_f, a_f)$$

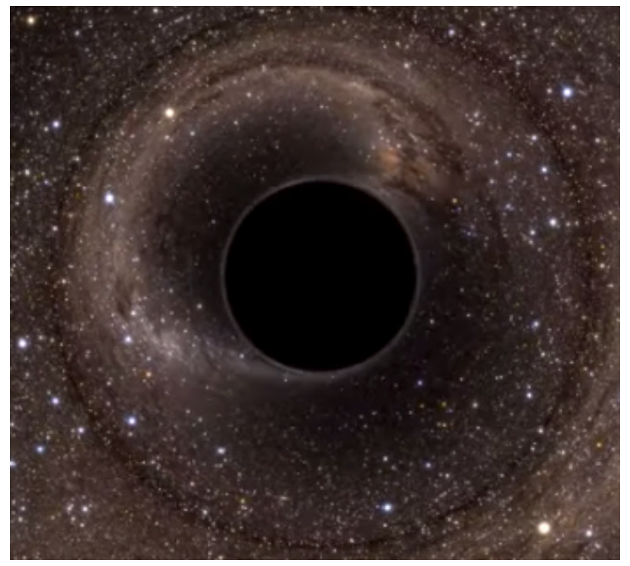


inspiralling phase
post-Newtonian theory

merger phase
numerical relativity

ringdown phase
BH perturbation theory





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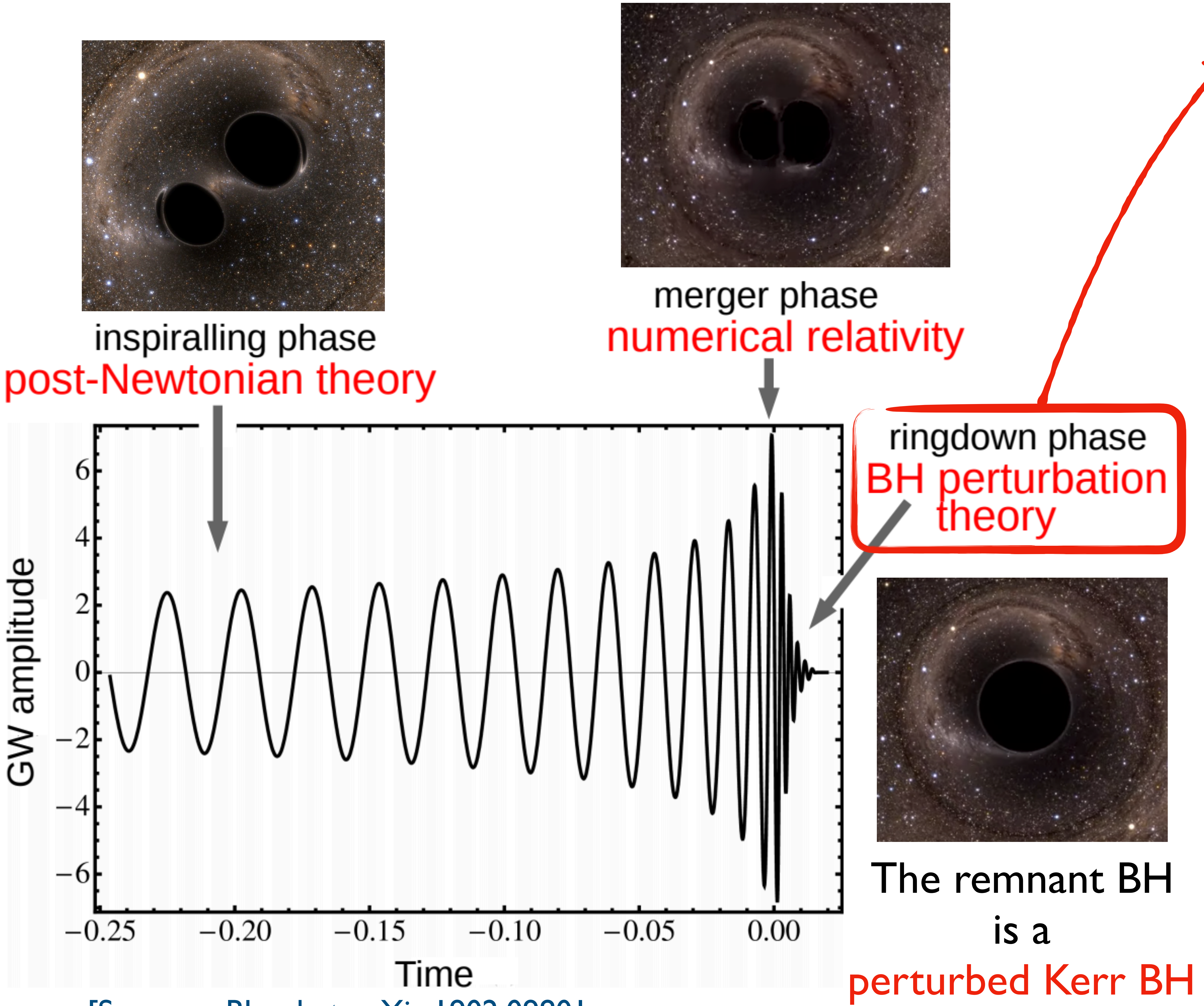

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- The signal is a superposition of damped sinusoids

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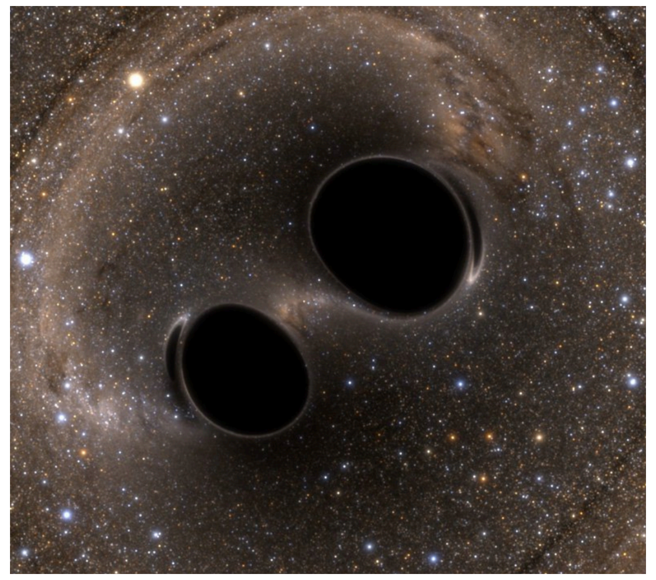

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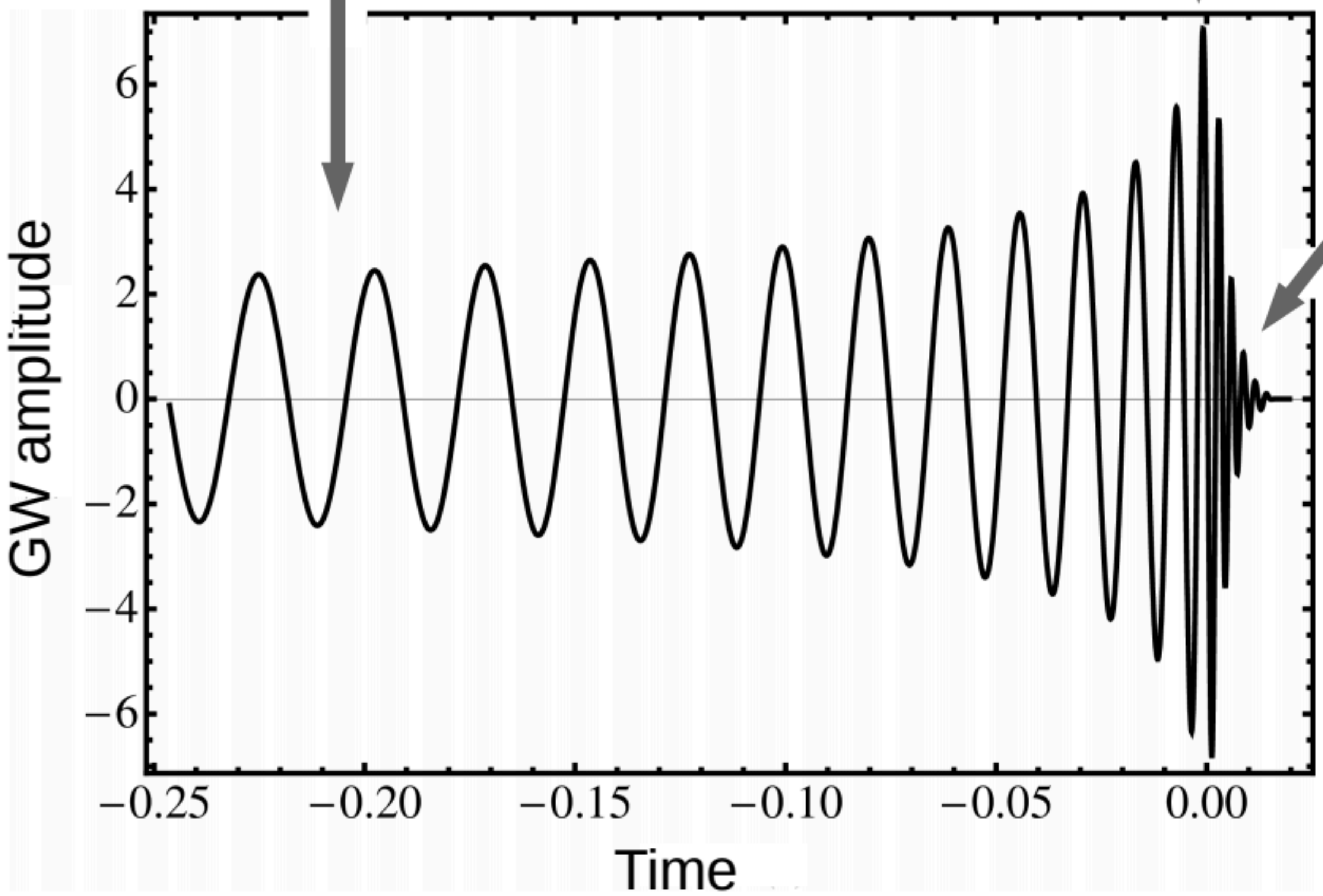

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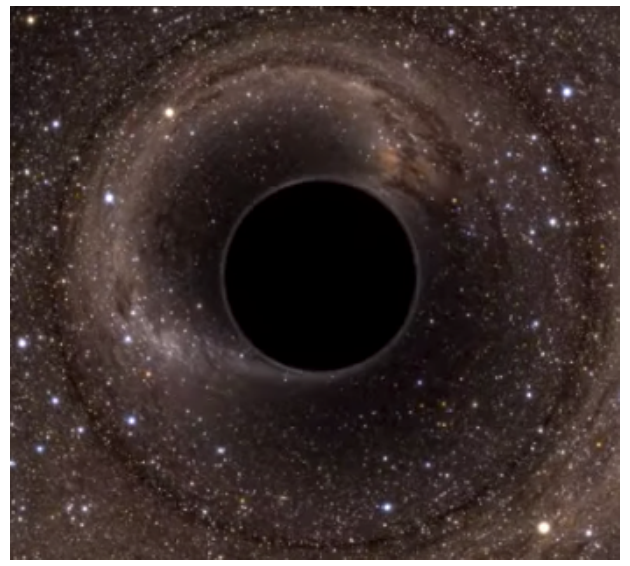

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- $\tilde{\omega}_{lmn}(M_f,a_f)$ are called quasinormal modes (QNMs)
- The signal is a superposition of damped sinusoids
- According to GR, there is a dominant QNM: $\tilde{\omega}_{220}$
- Thus:

$$h_+ - ih_\times \propto \exp[-i\omega_{220}t - t/\tau_{220}]$$

- A GW detector is sensitive to the detector strain:

$$h(t) = \mathcal{F}_+(\alpha',\delta',\psi)h_+(t) + \mathcal{F}_\times(\alpha',\delta',\psi)h_\times(t)$$

[Thorne, *300 Years of Gravitation*, CUP, Cambridge (1987)]

INTRODUCTION (II): TOOLS

- Bayes' theorem:

Posterior | Prior | Likelihood

$$p(\vec{\theta}|d,\mathcal{H}_i,I) = \frac{p(\vec{\theta}|\mathcal{H}_i,I)\,p(d|\vec{\theta},\mathcal{H}_i,I)}{p(d|\mathcal{H}_i,I)}$$

Evidence

$$\vec{\theta} = \{M_f, a_f, D_L, \ldots\}$$
$$d(t) = n(t) + h(t)$$
$$\mathcal{H}_i = \{\text{our assumed model}\}$$
$$I = \{\text{our prior information}\}$$

We can use this probability density function to determine estimators (e.g. median value) and credible intervals (e.g. 90% CI) for any of the waveform parameters (e.g. mass, spin, distance, etc.)

$$p(d|\mathcal{H}_i,I) = \int d\theta_1 \cdots d\theta_N \, p(d|\vec{\theta},\mathcal{H}_i,I)\,p(\vec{\theta}|\mathcal{H}_i,I).$$

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- Odds' ratio:

$$O^i_j = \frac{p(\mathcal{H}_i|d,I)}{p(\mathcal{H}_j|d,I)} = \frac{p(\mathcal{H}_i|I)}{p(\mathcal{H}_j|I)}\frac{p(d|\mathcal{H}_i,I)}{p(d|\mathcal{H}_j,I)} = \frac{p(\mathcal{H}_i|I)}{p(\mathcal{H}_j|I)} B^i_j$$

B^i_j → Bayes' Factor

TESTING BEKENSTEIN-MUKHANOV BLACK HOLES WITH RINGDOWN SIGNALS

A LONG-STANDING PROPOSAL

- Bekenstein & Mukhanov (for nonextremal black holes):

[Bekenstein, *Lett. Nuovo Cimento* 11, 467 (1974)]
[Mukhanov, *JETP Letters* 44, 63 (1986)]
[Kogan, *JETP Letters* 44, 267 (1986)]
[Garcia-Bellido, arXiv:hep-th/9302127 (1993)]
[Danielson, Schiffer, *PRD* 48, 4779 (1993)]
[Maggiore, *Nucl. Phys. B* 429, 205 (1994)]
[Bekenstein, Mukhanov, *Phys. Lett. B* 360, 7 (1995)]
[Lousto, *PRD* 51, 1733 (1995)]
[J. D. Bekenstein, *8th Marcel Grossmann Meeting*, Pts. A, pp. 92-111 (1997)]
[M. Maggiore, *PRL* 100, 141301 (2008)]
[Bekenstein, *PRD* 91, 124052 (2015)]
[...]

$$A_H^Q = \alpha l_P^2 N$$

where:

$$N \in \mathbb{Z}^+$$

$$l_P \approx 1.6 \times 10^{-35}\mathrm{m}$$

$$\alpha = \mathcal{O}(1)$$

- Some proposed values of α:
  - $\cdot\ \alpha = 8\pi \approx 25.1$ (Bekenstein) [Bekenstein, *PRD* 7, 2333 (1973)]
  - $\cdot\ \alpha = 8\ln 2 \approx 5.5$ (Davidson) [Davidson, *Int. J. Mod. Phys. D* 23, 1450041 (2014)]
  - $\cdot\ \alpha = 4\ln 3 \approx 4.4$ (Hod) [Hod, *PRL* 81, 4293 (1998)]
  - $\cdot\ \alpha = 4\ln 2 \approx 2.8$ (Mukhanov) [Mukhanov, *JETP Letters* 44, 63 (1986)]

OUR nGR RINGDOWN MODEL

- Foit & Kleban: heuristic interpretation of the BM conjecture

[Foit, Kleban, *CQG* 36 035006 (2019)]
[Cardoso, Foit, Kleban, arXiv: 1902.10164]

- MAIN CONSEQUENCE: the remnant BH settles down according to the "nGR" quantised QNM frequency:

$$\omega_1(M_f, a_f, \alpha) = \frac{1}{M_f G} \frac{\alpha\sqrt{1-a_f^2} + 16\pi a_f}{16\pi(1+\sqrt{1-a_f^2})}$$

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[Berti, Cardoso, Will, *PRD* 73, 064030 (2006)]

- "nGR" quantised QNM damping time derived from GR quality factor Q_{220}^{GR}:

$$\tau_1(M_f, a_f, \alpha) = 2\frac{Q_{220}^{GR}(M_f, a_f)}{\omega_1(M_f, a_f, \alpha)}$$

OUR FRAMEWORK

- Full time-domain analysis: RingdownTD [Carullo, Del Pozzo, Veitch, *PRD* 99, 123029 (2019)]
- Sampler: CPNest [Del Pozzo, Veitch, https://github.com/johnveitch/cpnest] [Skilling, *AIP Conference Proceedings*, 2004]
  - → Bayes' factors
  - → Posterior distributions of intrinsic and extrinsic parameters
    $\{M_f, a_f, \alpha, \mathcal{A}_{220}, \phi_{220}, \iota, \Phi, t_0\}$ $\{\alpha', \delta', D_L, \psi\}$
- Priors: uniform + $\alpha \in [0, 50]$
- Ringdown start time prior: $t_0 \in [10, 20] M_f$ after a fiducial GPS merger time t_M

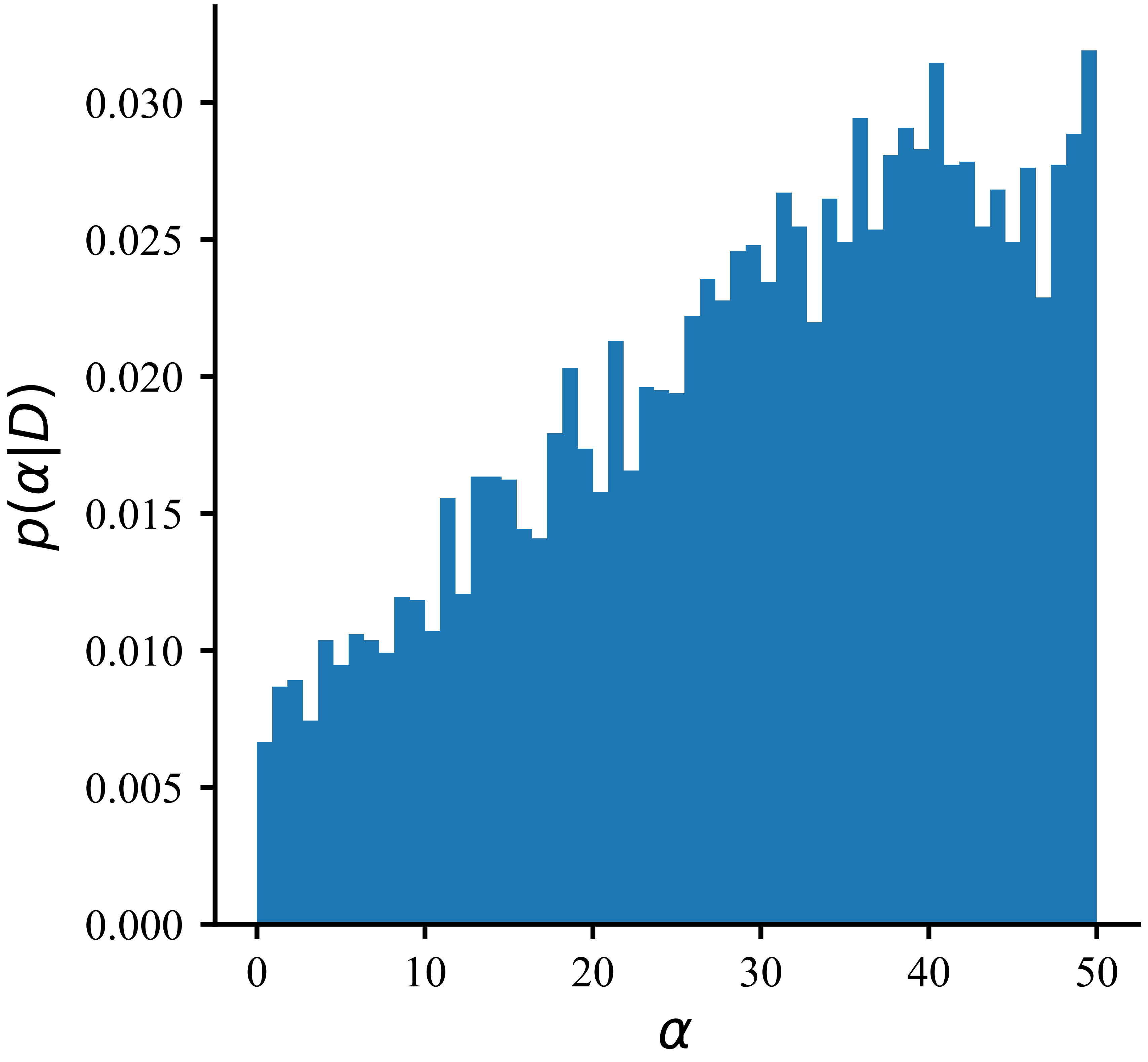
  $\simeq [3.5, 7.0]\mathrm{ms}$

  peak strain amplitude of $(h_+^2 + h_\times^2)(t)$

  [Abbott et al. (LVC), PRL 116, 221101 (2016)]

MEASURING α FROM GW150914

[DL, Carullo, Veitch, Del Pozzo, in Preparation]



Compare with:

[Foit, Kleban, *CQG* 36, 035006 (2019)]

Uniform priors:

$$M_f \in [10, 250] M_\odot$$

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$$\log B^{nGR}_{GR} = 0.1 \pm 0.1$$

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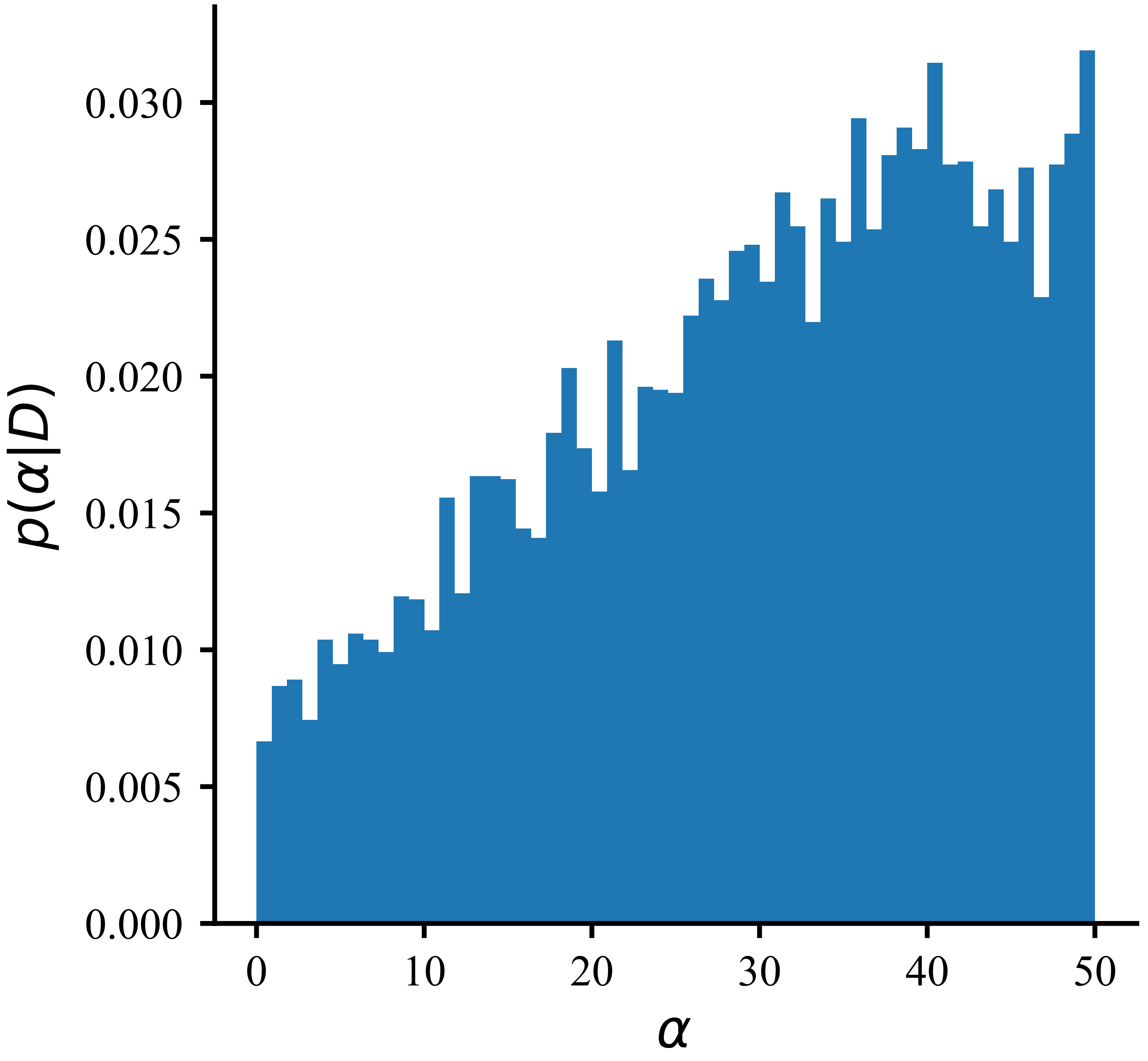

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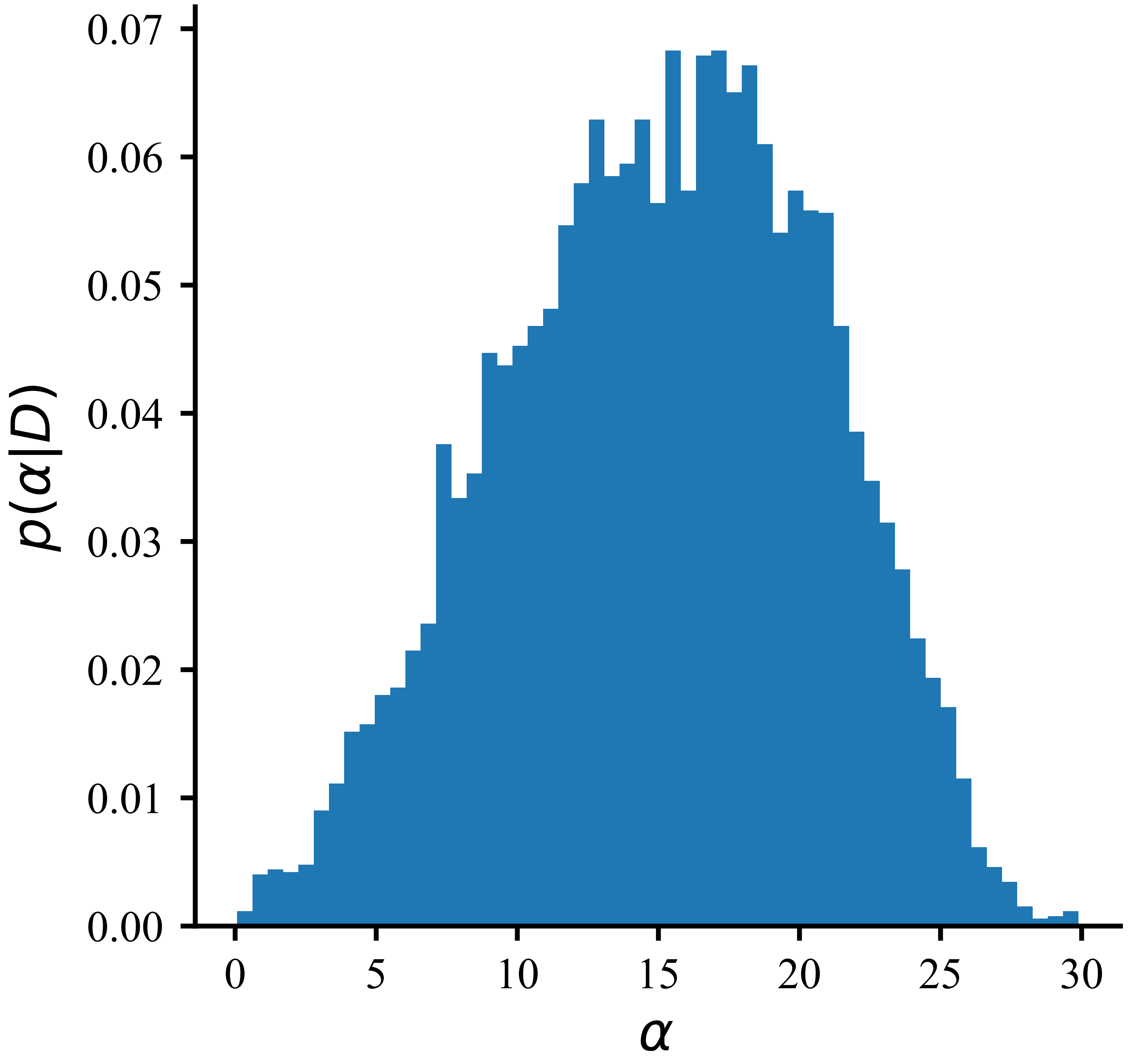

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LVC priors:

$$M_f \in [62, 75] M_\odot$$

$$a_f \in [0.55, 0.75]$$

$$\log B^{nGR}_{GR} = -1.6 \pm 0.1$$

SIMULATIONS

[DL, Carullo, Veitch, Del Pozzo, in Preparation]

- Single events in general do not provide much information about α

What about a population of GW150914-like events?

- Generate $\{\mathcal{A}_{220}, \phi_{220}\}$ using a non-precessing BBH ringdown model [London, arXiv: 1801.08208]
  - Injection: GR signal vs nGR signal

$$\omega_{220} = \omega_{220}(M_f, a_f) \qquad \omega_1 = \omega_1(M_f, a_f, \alpha)$$
$$\tau_{220} = \tau_{220}(M_f, a_f) \qquad \tau_1 = \tau_1(M_f, a_f, \alpha)$$

[Berti, Cardoso, Will, *PRD* 73, 064030 (2006)]

  - Recovery: GR template vs nGR template

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- We can make model selection: e.g. $\log B^{nGR}_{GR} = \log \frac{P(D|nGR)}{P(D|GR)}$

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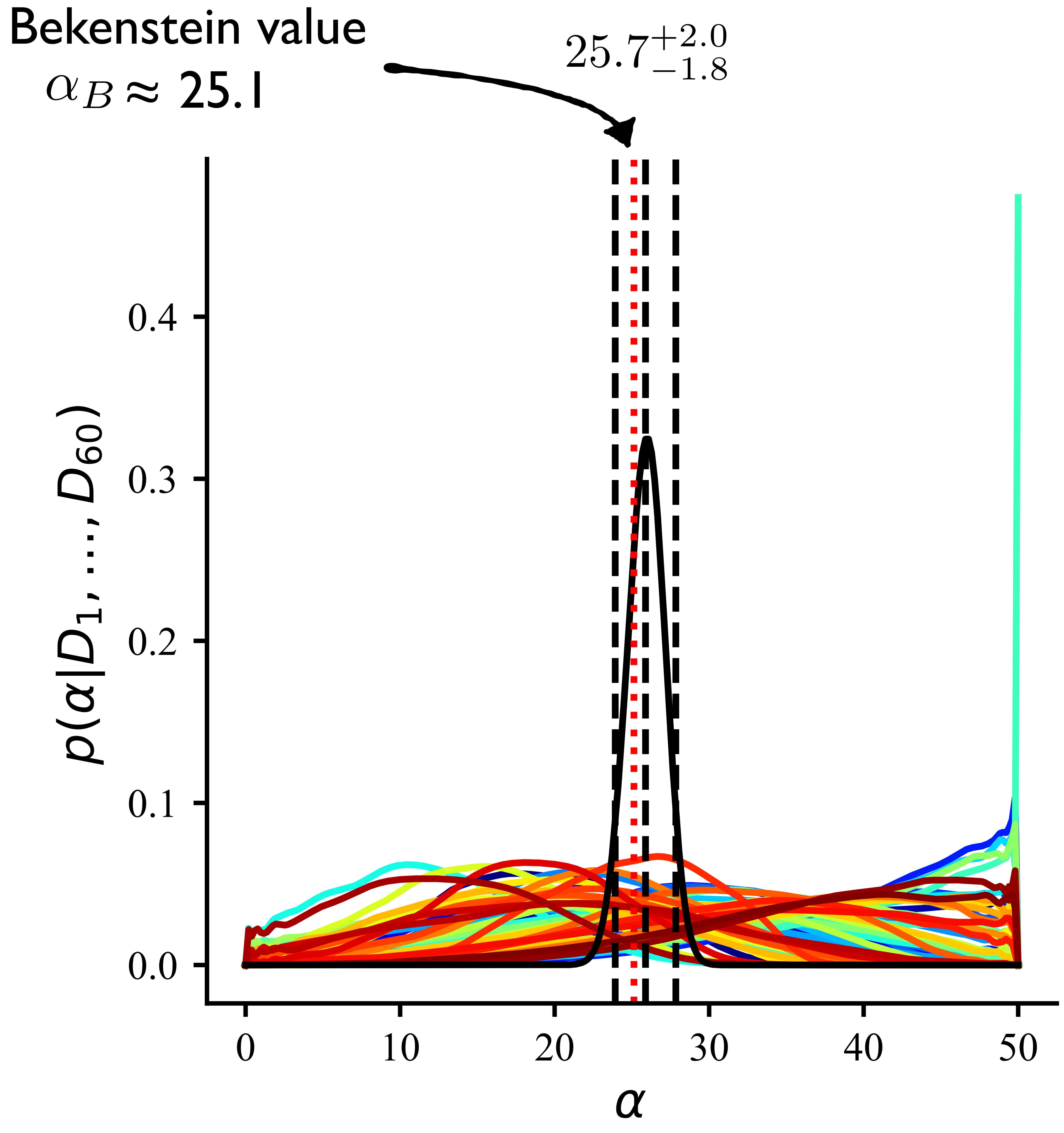
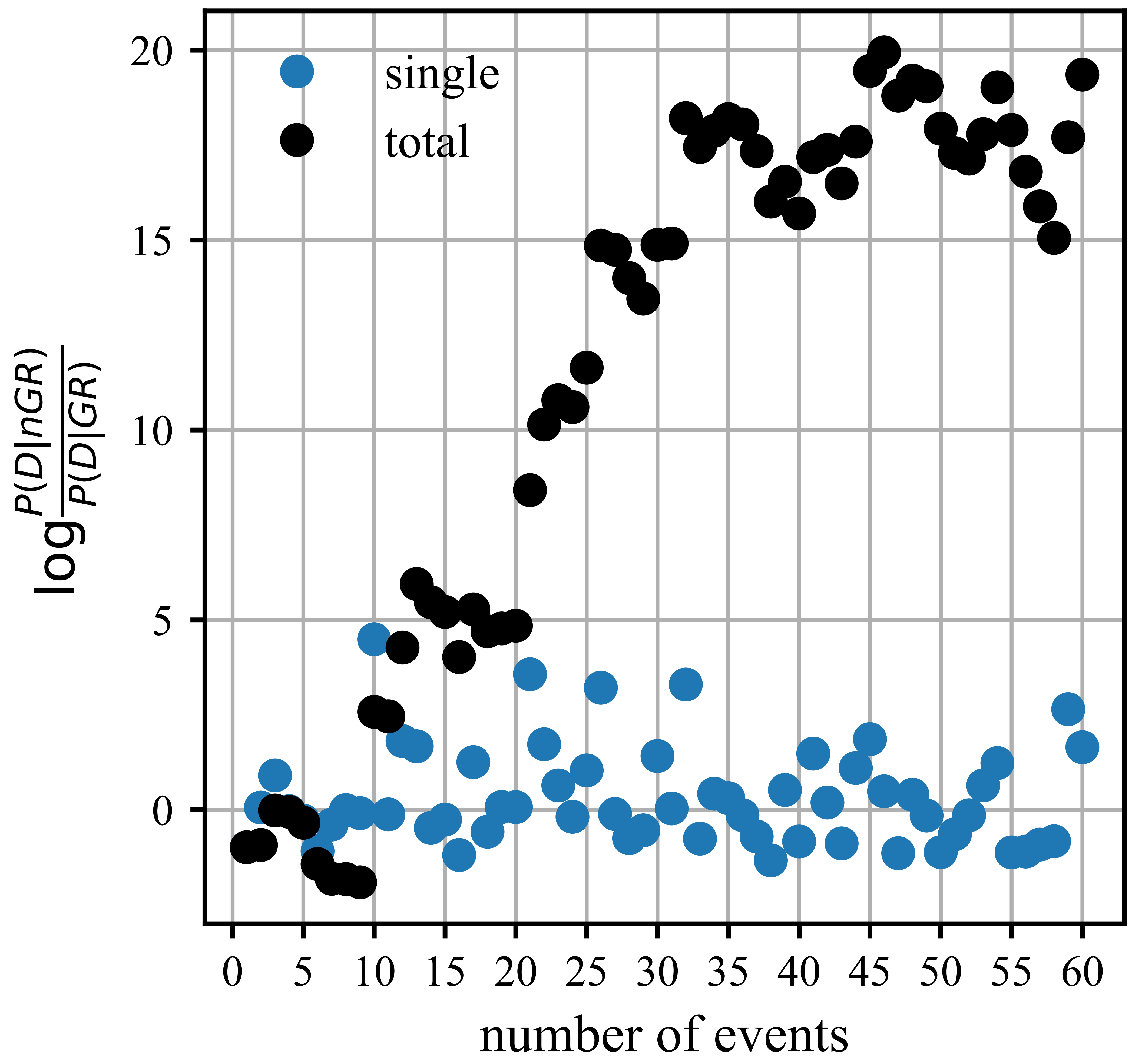
SIMULATIONS: nGR BEKENSTEIN BHs

[DL, Carullo, Veitch, Del Pozzo, in Preparation]





CONCLUSIONS (SO FAR) AND PROSPECTS

- We have an infrastructure to measure possible deviations from GR analysing the ringdown
- We have shown an application to the area quantisation conjecture
- If a theory predicts physically meaningful $\begin{array}{c}\omega_{nGR} = \omega_{nGR}(M_f, a_f, \vec{\theta}) \\ \tau_{nGR} = \tau_{nGR}(M_f, a_f, \vec{\theta})\end{array}$, we can test it on real data and explore its observational effects through simulated events

OUTLOOK

- Applying the method to all the GWTC-1 events
- Relaxing the assumption on the GR quality factor
- Assessing stealth biases due to non-GR effects
- Article in preparation

BLACK HOLE RINGDOWN MODELS WITH OVERTONES

RINGDOWN: THE PARADIGM

- The merger phase is highly nonlinear
- The ringdown phase is linear

What is the time of transition between the nonlinear-linear regime?

- Moving the ringdown start time can drastically alter parameter inference

[Abbott et al (LVC), *PRL* 116, 221101 (2016)]

- Main desiderata of a ringdown template:
  - fit the detected ringdown waveform correctly
  - infer the fiducial (IMR) final mass and spin of the remnant BH

When does ringdown begin?

RINGDOWN: A NEW PROPOSAL

- The QNM spectrum $\tilde{\omega}_{lmn}$ is characterised by 3 numbers (l, m, n)
- For a given (l, m):
  - the $n = 0$ is the longest-lived mode (the "fundamental")
  - the $n \geq 1$ die out very fast and are called "overtones"

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Giesler et al. suggestion:

[Giesler, Isi, Scheel, Teukolsky, arXiv:1903.08284]

- Start the linear description *at* (not after!) the merger time t_M

AND

- Add N *overtones* of the dominant harmonic

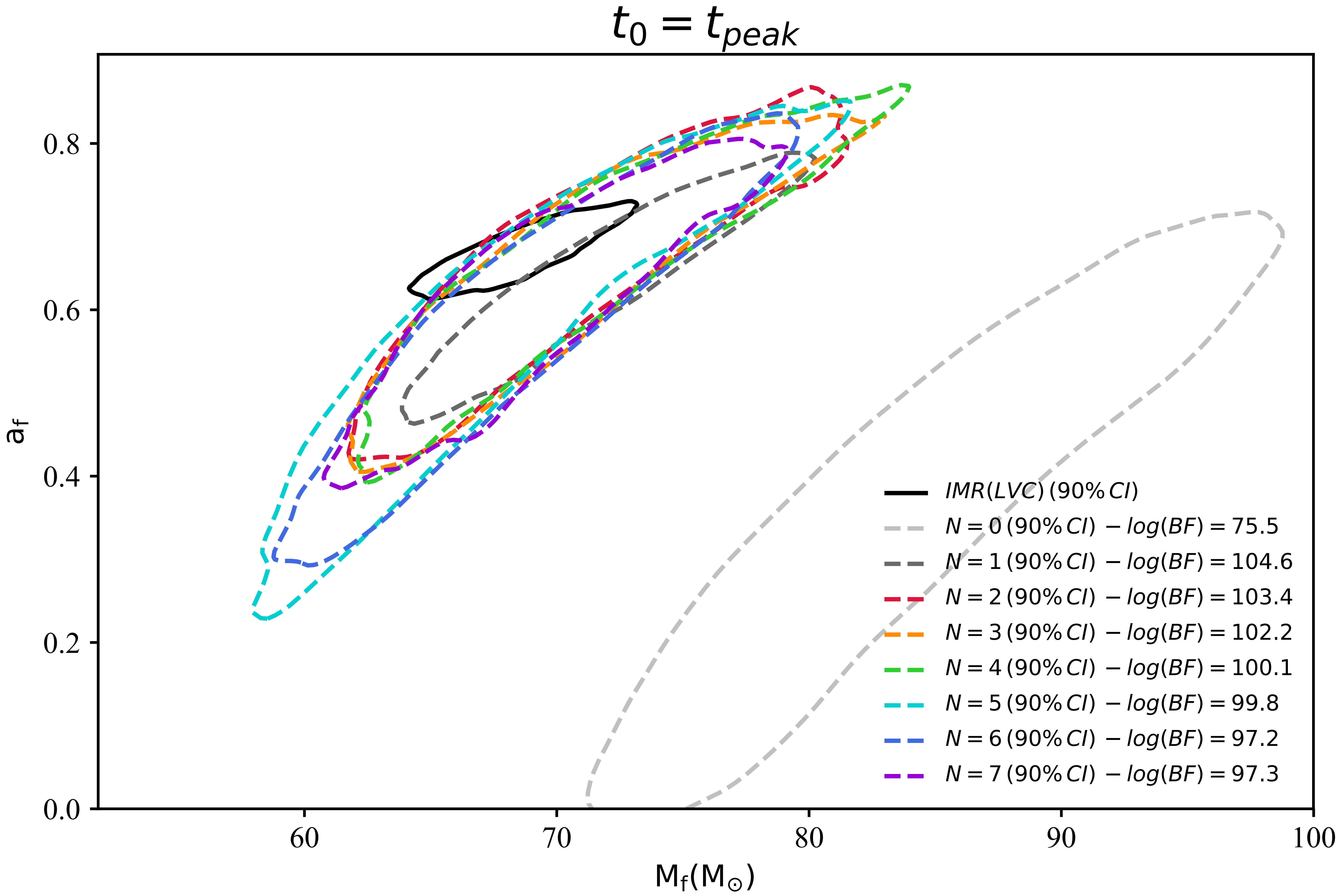
$$h_{lm}^{N}(t) = \frac{M_f}{D_L} \sum_{n=0}^{N} \left\{ \tilde{A}_{lmn} \, {}_{-2}Y^{lm}(\iota, \Phi) e^{i(t - t_{\rm peak})\tilde{\omega}_{lmn}} + \text{c.c.} \right\} \qquad t \geq t_{\rm peak} \equiv t_M$$

How many overtones do we need?

- This question can be naturally answered with Bayesian model selection

RESULTS (I): GW150914 WITH OVERTONES

- Substantial support for the model, although no full agreement with the results obtained by the same authors on the same data [Isi, Giesler, Will, Scheel, Teukolsky, *PRL* 123, 111102 (2019)]



RESULTS (II): TESTING GR

- Test of the no-hair theorem following the phenomenological approach presented in Li et al. [Li, Del Pozzo, Vitale, Van Der Broeck, Agathos, Veitch, Grover, Sidery, Sturani, Vecchio, *PRD* 85, 082003 (2012)]
- Define $\mathcal{H}_{modGR}$ with $\omega_{eff}(M_f, a_f) = (1 + d\omega)\,\omega_{GR}(M_f, a_f)$ and calculate $O_{GR}^{modGR} \equiv \frac{P(\mathcal{H}_{modGR}|d,I)}{P(\mathcal{H}_{GR}|d,I)}$ for each overtone model (N=1,2,…)
- Our results:

$$N = 1: \quad {}^{(2)}\mathcal{O}_{GR}^{modGR} = 1.00$$

$$N = 2: \quad {}^{(3)}\mathcal{O}_{GR}^{modGR} = 0.76$$

$$N = 3: \quad {}^{(4)}\mathcal{O}_{GR}^{modGR} = 0.64$$

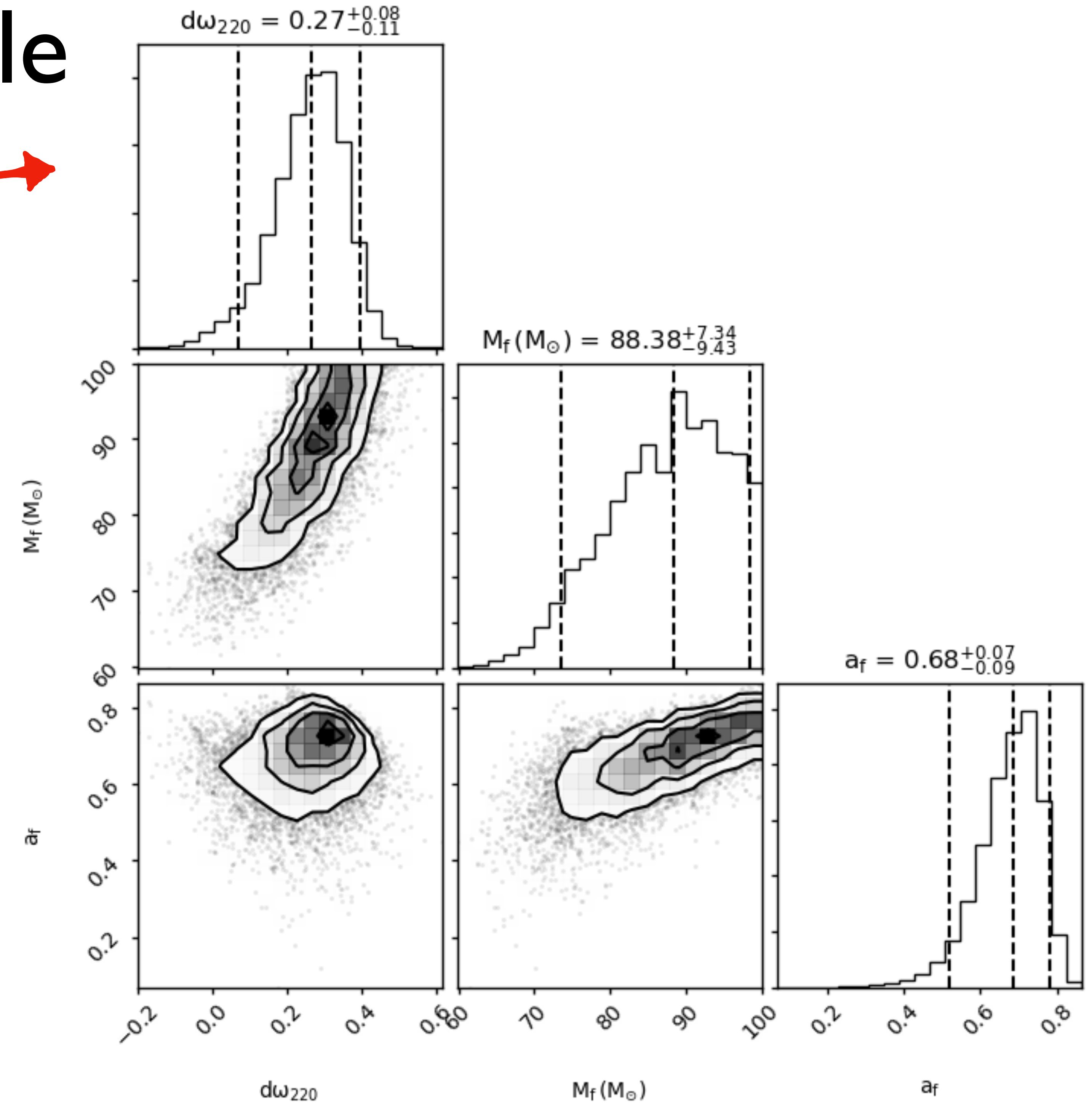
No preference towards overtones using GW150914

(PRELIMINARY) CONCLUSIONS

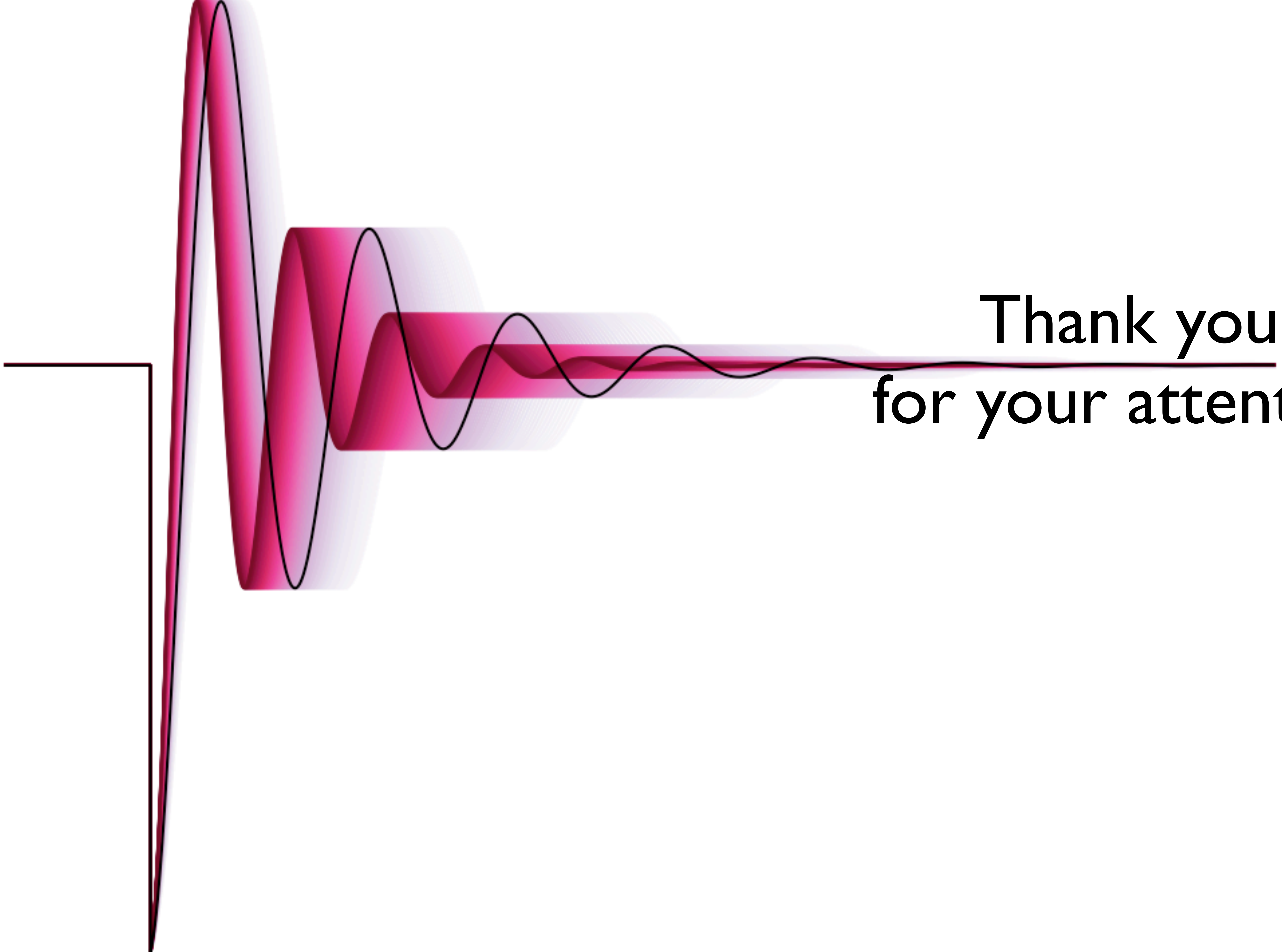
- Models with 1<N<3 seem to give reliable parameter inference
- Tests of GR seem possible: no evidence of violation of the no-hair theorem following a logB study

BUT

- The inclination ι is not consistent with a full IMR waveform analysis
- The measure of the parameter of violation $d\omega$ is not stable



Further investigations are ongoing!



Thank you
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