Topology and θ-dependence in QCD and QCD-like theories

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PhD student:
Claudio Bonanno

Supervisor:
Prof. Massimo D'Elia

Talk overview

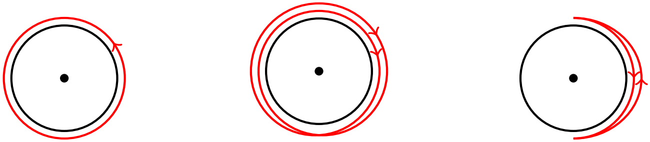
- Brief review of θ-dependence and of its phenomenological relevance,
- Results for θ-dependence of $2d$ CP^{N-1} models in the large-N limit (CB, Bonati, D'Elia, 2018; Berni, CB, D'Elia, 2019),
- Topology via spectral projectors with staggered fermions (CB, Clemente, D'Elia, Sanfilippo, 2019),
- Future perspectives and plan for third year.

The topological charge in $4d$ gauge theories

The topological charge of the gluon field A_μ is

$$Q = \frac{1}{64\pi^2} \int d^4x \, \epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x) \in \mathbb{Z}.$$

It can be interpreted as the number of windings of A_μ around the group manifold.



$Q = 1$ $\quad$ $Q = -2$ $\quad$ $Q = 0$

The topological charge can be coupled to the ordinary QCD action via the new parameter θ:

$$S_{\mathrm{QCD}} \to S(\theta) = S_{\mathrm{QCD}} - i\theta Q.$$

The strong-CP problem

A CP transformation (Charge Conjugation + Parity) changes $S(\theta)$:

$$Q \underset{CP}{\rightarrow} -Q \implies \theta \underset{CP}{\rightarrow} -\theta.$$

If $\theta \neq 0 \implies$ QCD explicitly breaks the CP symmetry! In principle, θ can assume any value in $[0, 2\pi)$.

So far no experimental evidence of strong CP violation has been found $\implies \theta_{exp}$ is compatible with 0.
The most stringent upper bound comes from the measure of the neutron electric dipole moment (which vanishes if CP is preserved):

$$d_n^{theo} \sim \theta \left(10^{-16}\ e \cdot \text{cm}\right), \qquad |d_n^{exp}| \lesssim 10^{-26}\ e \cdot \text{cm},$$

$$\implies |\theta_{exp}| \lesssim 10^{-10}.$$

Topology, θ-dependence and axion phenomenology

Axion (Peccei, Quinn, 1977; Weinberg, 1978; Wilczek, 1978) is a new light pseudo-scalar particle introduced to solve the strong-CP problem. Axion physics is tiedly connected to the θ-dependence of the vacuum energy density:

$$f(\theta) = \frac{1}{2}\chi\theta^2\left(1 + \sum_{n=1}^{\infty} b_{2n}\theta^{2n}\right),$$

$$\chi = \left.\frac{\langle Q^2\rangle}{V}\right|_{\theta=0}, \qquad b_2 = -\frac{1}{12}\left.\frac{\langle Q^4\rangle - 3\langle Q^2\rangle^2}{\langle Q^2\rangle}\right|_{\theta=0},$$

since

$$m_a^2 \sim \chi, \quad V_{eff}(a) \sim f(\theta).$$

Being topological properties non-perturbative, the lattice is the only fully non-perturbative first-principle tool to investigate θ-dependence.

The $2d$ CP^{N-1} models

The $2d$ CP^{N-1} models are toy models of QCD which share many features with it. In this case θ-dependence is analytically calculable in the large-N limit.

$$S(\theta) = \frac{N}{g} \int d^2x |(\partial_\mu + iA_\mu)z|^2 - i\theta Q, \;\; Q = \frac{1}{2\pi}\epsilon_{\mu\nu} \int d^2x F_{\mu\nu}(x).$$

Relevant properties:

- color confinement ($z \sim$ quarks, $A_\mu \sim$ gluon, $N \sim N_c$),
- asymptotic freedom,
- existence of topological charge Q and θ-dependence.

Being them cheaper than QCD and analytically solvable, they have also been studied on the lattice as test-beds to validate numerical methods to be used in lattice QCD.

θ-dependence of $2d$ CP^{N-1} models in the large-N limit

Motivations

For $2d$ CP^{N-1} models, the θ-dependence of the vacuum energy is known in the large-N limit ($1/N$ expansion):

$$f(\theta) = \frac{1}{2}\chi\theta^2\left(1 + \sum_{n=1}^{\infty} b_{2n}\theta^{2n}\right),$$

$\xi^2\chi = \frac{1}{2\pi}\frac{1}{N} - 0.0605\frac{1}{N^2} + O\left(\frac{1}{N^3}\right)$ (D'Adda et al., 1978; Rossi et al., 1991),

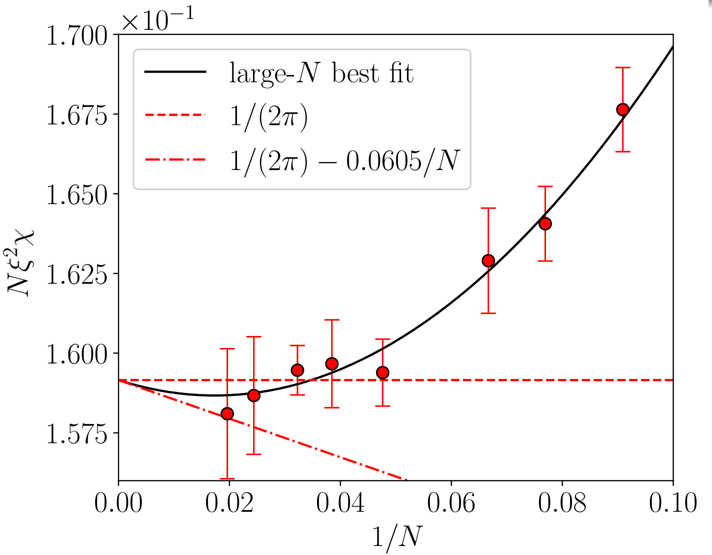
$b_2 = -\frac{27}{5}\frac{1}{N^2} + O\left(\frac{1}{N^3}\right)$ (Del Debbio et al., 2006).

Lattice simulations have failed to confirm these results, aside from the leading-order term of $\xi^2\chi$:

- lattice data seem to point out a small and **positive** next-to-leading correction to $\xi^2\chi$,
- the leading-order coefficient of b_2 from the lattice appears to be larger of about a factor 2.

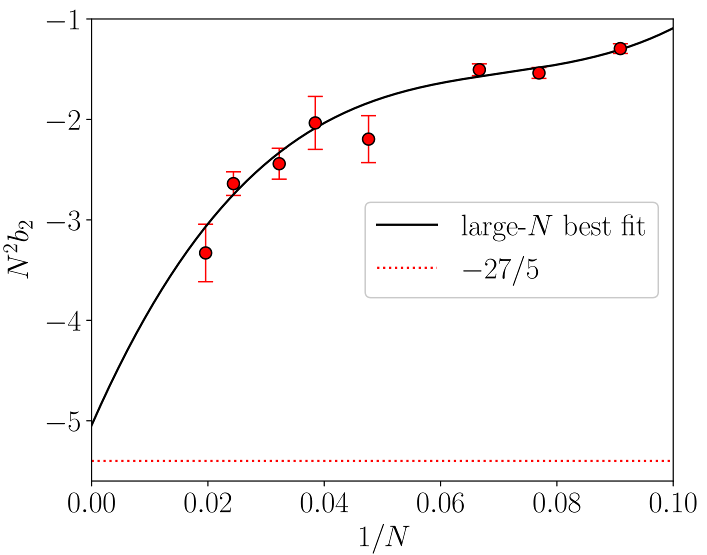
Goal: improve lattice validation of theoretical results.

Large-N behavior of $N\xi^2\chi$



$$(N\xi^2\chi)_{fit} = 1/(2\pi) - 0.057(12)(1/N) + 1.8(4)(1/N)^2$$

Large-N behavior of $N^2 b_2$



$$\bar{b}_2 \equiv \lim_{N\to\infty} N^2 b_2 = -27/5 = -5.4\,, \qquad (\bar{b}_2)_{fit} = -5.0(1.1)$$

Topology via spectral projectors with staggered fermions

Motivations

The QCD axion, being weakly coupled to the Standard Model, is a good Dark Matter candidate.
In this context, the behavior of the axion effective potential at high temperatures is extremely relevant to access today axion relic abundance and mass, which are also important for experimental researches.

These quantities are related to QCD topological observables:

$$m_a^2 \sim \chi, \quad V_{\mathit{eff}}(a) \sim f(\theta).$$

This constitutes a strong motivation to study the high-T behavior of χ and of θ-dependence in QCD.

Numerical problems - 1

In the QCD path-integral, field configurations are weighted with the determinant of the Dirac operator:

$$\det\{\not{D} + m_q\} = \prod_{\lambda \in \mathbb{R}} (i\lambda + m_q)\,.$$

The Index Theorem relates the presence of zero-modes in the spectrum of $\not{D}$ to the topological charge of the gluon field:

$$Q = \mathrm{Index}\{\not{D}\} = \mathrm{Tr}\{\gamma_5\} = n_+ - n_-.$$

If a configuration has $Q \neq 0$, lowest eigenvalues are $\lambda_{min} = m_q$.

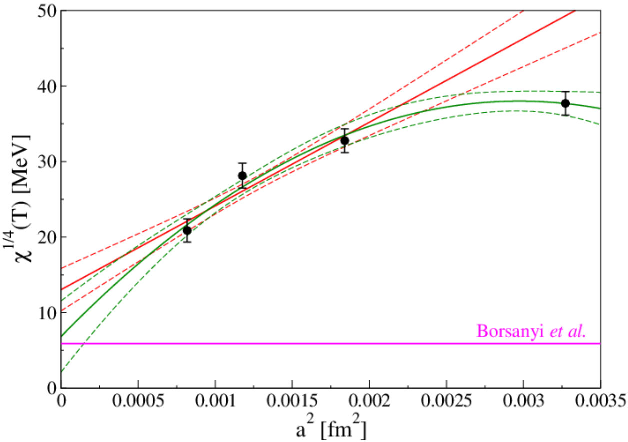
On the lattice, however, most of the discretizations of the Dirac operator do not have exact zero-modes.

$\Longrightarrow$ Wrong suppression of configurations with non-zero charge!

$$\lambda_{min} = m_q \longrightarrow m_q + i\lambda_0(a), \qquad \lambda_0(a) \underset{a\to 0}{\to} 0.$$

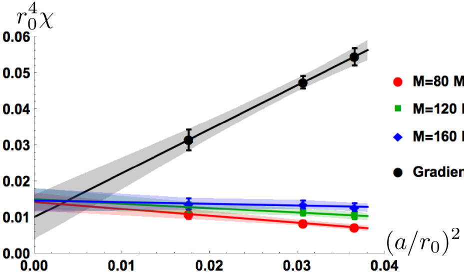
Numerical problems - 2

This result in large discretization errors $\Longrightarrow$ extrapolation towards the continuum not under control (Bonati et al., 2018):



In (Borsanyi et al., 2016) a continuum estimation of χ at high-T has been obtained by reweighting configurations with the continuum lowest eigenvalues by hand.

A possible solution could be to use a lattice version of the Index Theorem and define a fermionic topological charge. Using the same "bad" operator to weight configurations and to count eigenmodes to measure Q may improve errors.



Idea supported by results at $T = 0$ (Alexandrou et al., 2017): Wilson fermions employed for the path-integral and for the measure of χ through spectral projectors $\longrightarrow$ improved scaling of χ towards the continuum!

Goal: extension of spectral projectors to staggered fermions in view of an application to ongoing high-T QCD simulations with same fermionic discretization.

Spectral projectors for staggered fermions

In the continuum, only zero-modes contribute to Q. This is not true on the lattice, due to the absence of exact zero-modes:

$$Q = \mathrm{Tr}\{\gamma_5\} \longrightarrow \mathrm{Tr}\{\gamma_5 \mathbb{P}_M\},$$

$$\mathbb{P}_M = \textstyle\sum_{|\lambda_k| \leq M} u_k u_k^\dagger, \qquad \not{D}_{stag} u_k = i\lambda_k u_k.$$

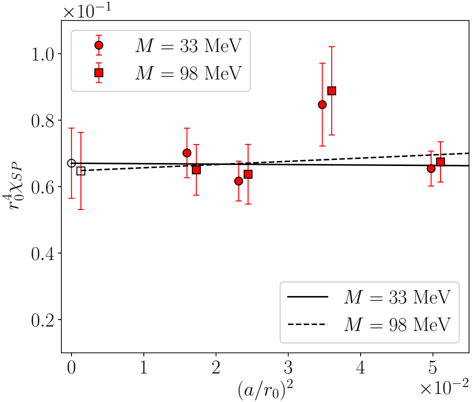
Dirac degrees of freedom are "staggered" on hypercubes:
1 staggered fermion $\equiv N_f = 16/4 = 4$ flavors
$\implies$ Degeneration in the spectrum $\implies$ Mode over-counting!

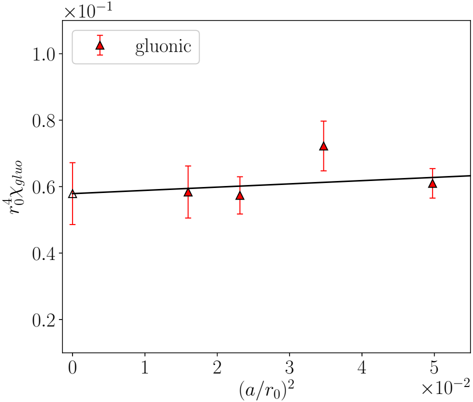
$$\implies Q_L = \frac{1}{N_f} \mathrm{Tr}\{\gamma_5 \mathbb{P}_M\}.$$

Lattice charge gets also a renormalization Z_Q, which can be derived from Ward identities for the flavor-singlet axial current:

$$Q_R = Z_Q Q_L, \qquad Z_Q^2 = \frac{\langle \mathrm{Tr}\{\mathbb{P}_M\}\rangle}{\langle \mathrm{Tr}\{\gamma_5 \mathbb{P}_M \gamma_5 \mathbb{P}_M\}\rangle}.$$

Spectral projectors are tested on the pure-gauge theory, where gluonic charge is used as a benchmark to validate the method.





(CB, Clemente, D'Elia, Sanfilippo, 2019)	Stag. spectral proj. $M = 33$ MeV	Stag. spectral proj. $M = 98$ MeV	Gluonic definition
$r_0^4 \chi_{\text{YM}} \cdot 10^3$	67(11)	65(12)	58(9)

Conclusions and future outlooks

In the pure-gauge case, spectral projectors and the gluonic definition yield similar lattice corrections to the continuum limit. In QCD the situation is very different:

- Bad suppression of lowest eigenmodes of the Dirac operator results in large lattice corrections for the gluonic charge,
- spectral projectors with Wilson fermions and at $T = 0$: fermionic topological susceptibility has improved scaling towards the continuum compared to the gluonic one.

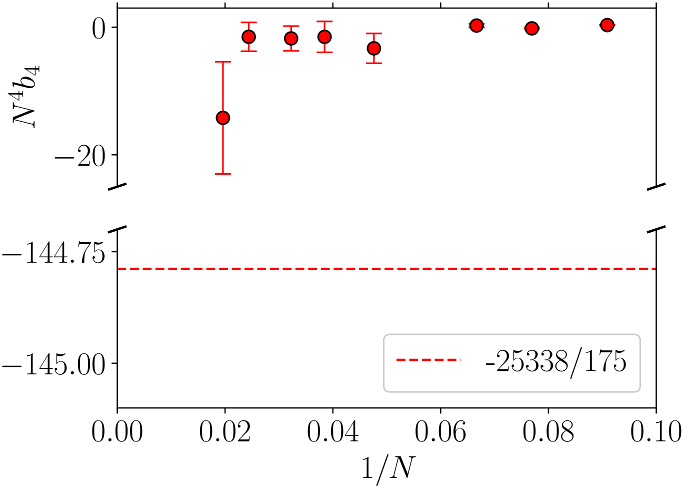
Plan for next year

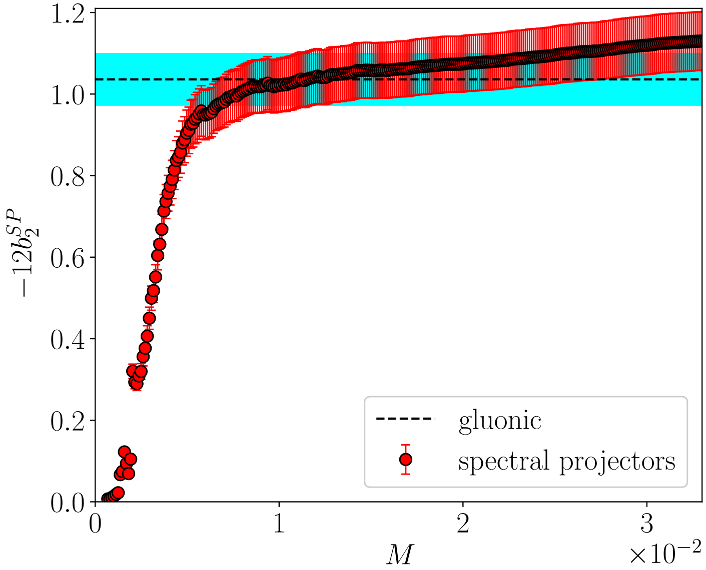
Apply spectral projectors to ongoing simulations of QCD with staggered fermions to improve lattice studies of θ-dependence at high temperatures, relevant for axion cosmology at early times.

Thank you for your attention!

Back-up slides

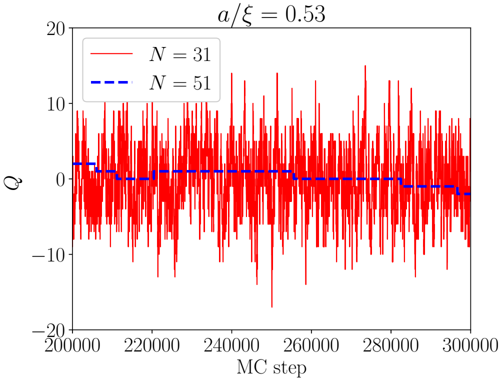
Large-N behavior of b_4 - 2d CP^{N-1} models




1.2
1.0
0.8
0.6
0.4
0.2
0.0
$-12b_2^{SP}$
0
1
2
3
M
$\times 10^{-2}$
gluonic
spectral projectors

Critical Slowing Down of topological modes

Serious computational problem to face to study the large-N limit of topological observables. Standard algorithms use local updates to change the topological charge $\implies$ as $a \to 0$ it becomes harder and harder to change Q $\implies$ fluctuations of Q become rare and the charge evolution freezes!



Critical Slowing Down worsens rapidly as N is increased, appearing already at large values of a $\implies$ large values of N become rapidly not feasible with standard techniques!

Consider a collection of lattice copies with different boundary conditions, smoothly going from from periodic to open ones. Each replica has an independent time evolution and different copies are swapped from time to time. Charge is quickly changed in the opened replica and then passed to the periodic copy via swaps (Hasenbusch, 2018).