Heat propagation in superfluid Helium-4

Marco Barsanti, PhD student
University of Pisa

marco.barsanti@phd.unipi.it

Outline

- Introduction to superfluidity
- Heat propagation:
    - Two-fluid model
    - Entropy waves
- Conclusions

Introduction to superfluidity

Helium-4 → Weak interactions

→ Boson

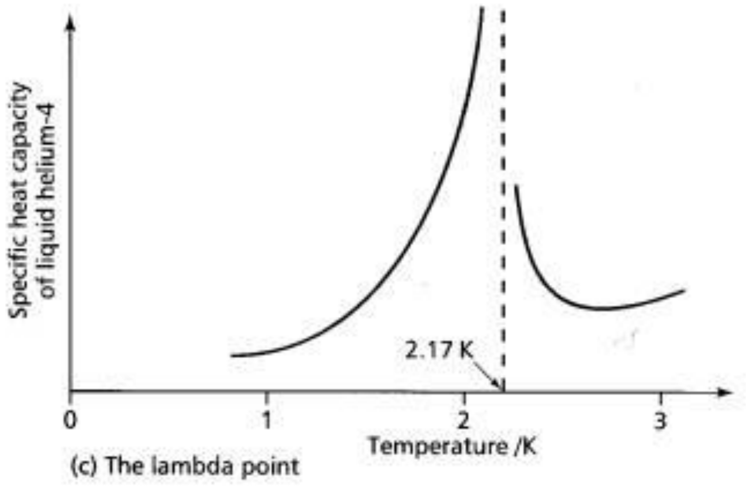


Fig. 11.6E Liquid helium as a superfluid

Phase transition at T=2.17 K

Fluid → Superfluid

Fluid → Superfluid

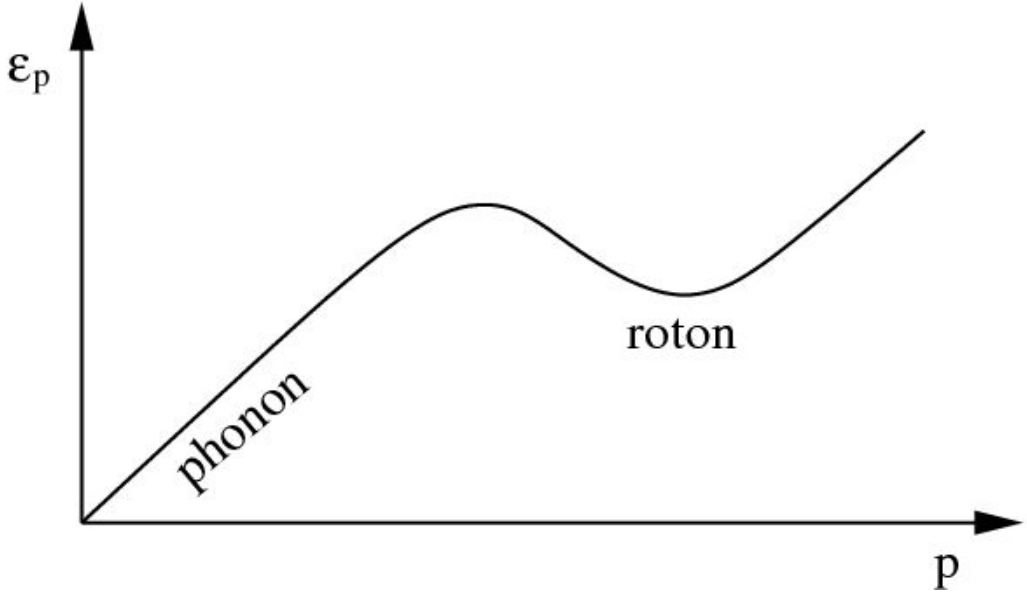
- Very low viscosity
- Quantum vortices
- High thermal conductivity

Energy spectrum

$$H = E_0 + \sum_p \varepsilon(p)\, b_p^\dagger b_p$$

“condensation energy”

“collective excitations”



“Any spectrum in which sufficiently small excitations are phonons will lead to superfluidity.”

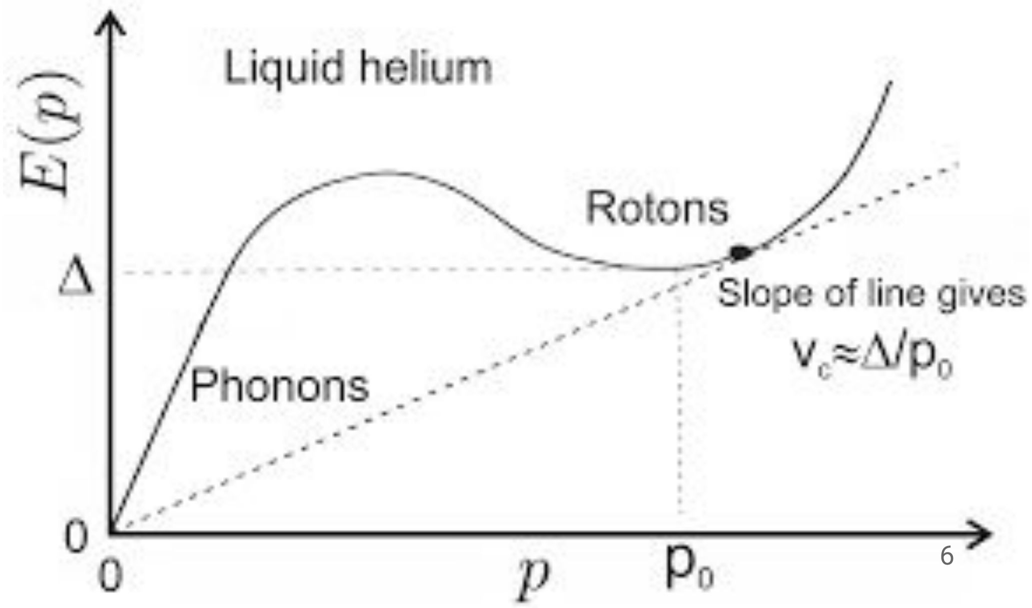
L.Landau, *Statistical Physics, Vol.9*

No Viscosity at T=0

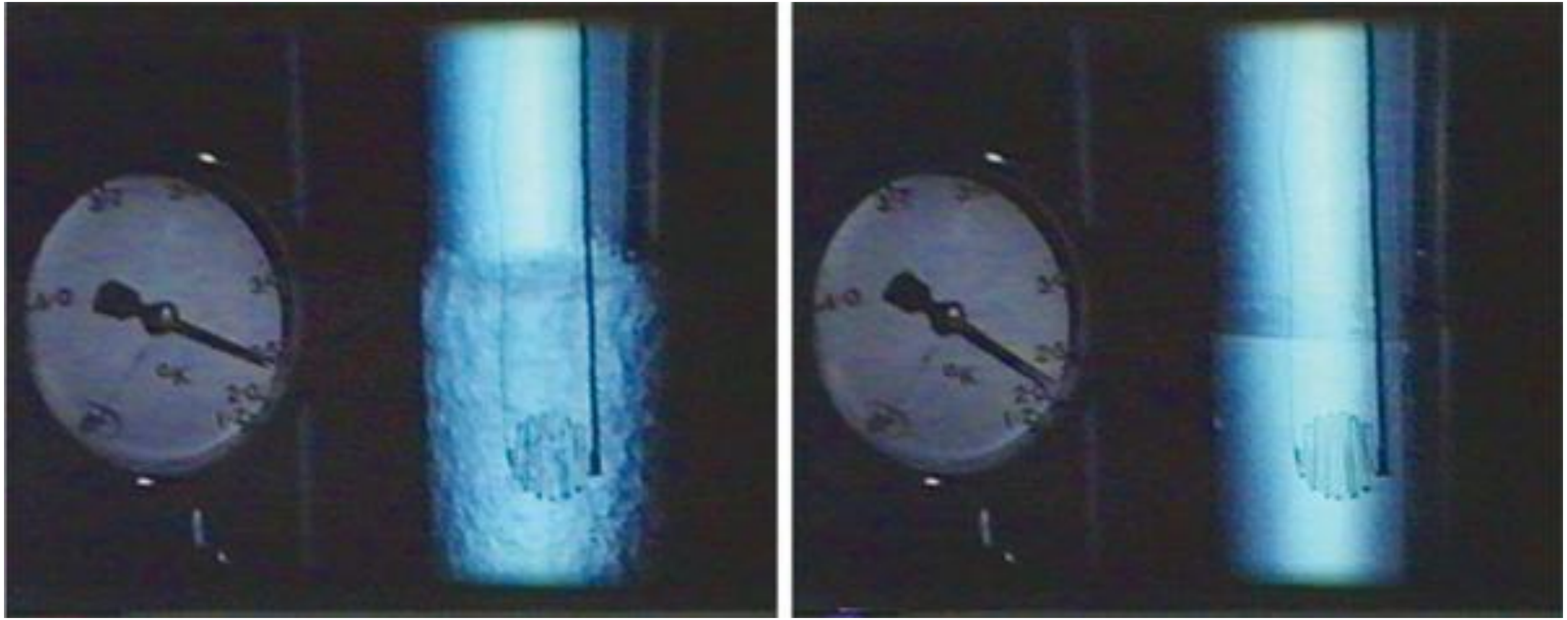
No friction for

$$v < v_c$$

$$v_c \equiv Min\left[\frac{\varepsilon(p)}{p}\right]$$



Heat propagation



Stop boiling after phase transition

"Sound wave" propagation of heat ➡ High conductivity

Theoretical approach

Two-fluid model

" Theory of the Superfluidity of Helium II "
L.Landau, Phys. USSR, 5:71, 1941

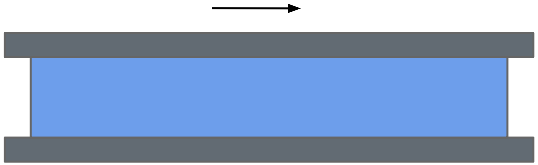
Not trivial fluid dynamics

Sound waves
+
Entropy/Temperature waves

Two-fluid model

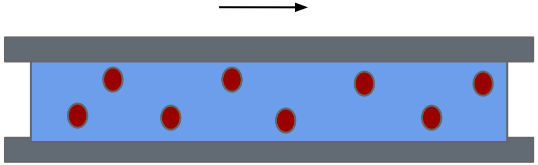
$v < v_c$

T=0



$v < v_c$

T>0



Gas of free bosons moving at **v**

$$\mathbf{P} = \int \mathbf{p}\, n(\varepsilon(p) - \mathbf{p}\mathbf{v})\, \frac{d^3p}{h^3}$$

$$= \mathbf{v}\,\rho_n \qquad \rho_n < \rho$$

$\rho_n = \rho_n(T)$ “normal component”

Two-fluid model

$$\rho = \rho_s + \rho_n$$

$$\mathbf{J} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n$$

Superfluid component

- $T = T_c \quad \rho_s = 0$
- No viscosity
- No entropy

Normal component

- $T = 0 \quad \rho_n = 0$
- Viscosity
- Entropy of the "excitations" gas

Fluid dynamic equations at O(v)

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$

Continuity equation (mass)

$$\frac{\partial \vec{J}}{\partial t} + \vec{\nabla} P = 0$$

Momentum equation

$$\frac{\partial (\rho s)}{\partial t} + \vec{\nabla}(\rho s \vec{v}_n) = 0$$

Continuity equation (entropy)

$$\frac{\partial \vec{v}_s}{\partial t} + \vec{\nabla} \mu = 0$$

$$\vec{\nabla} \times \vec{v}_s = 0$$

Entropy waves

Perturbative expansion

$$\rho = \rho_0 + \rho', \quad s = s_0 + s', \quad T = T_0 + T' \quad etc.$$

$$\frac{\partial^2 \rho'}{\partial t^2} = u_1^2 \nabla^2 \rho' \qquad u_1 = \sqrt{\left(\frac{\partial \rho}{\partial P}\right)_s}$$

First sound

$$\frac{\partial^2 s'}{\partial t^2} = u_2^2 \nabla^2 s' \qquad u_2 = \sqrt{\left(\frac{T s^2 \rho_s}{\rho_n c}\right)_p}$$

Second sound

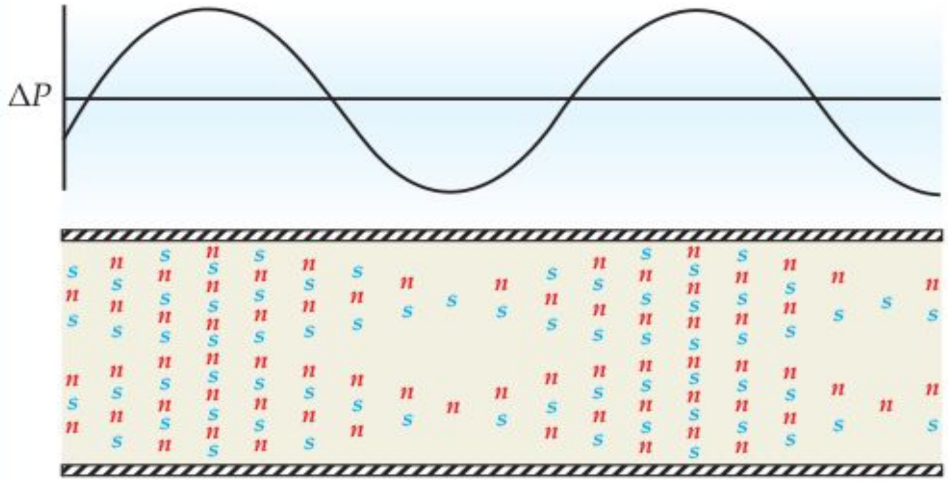
Entropy/Temperature

$$s' = \left(\frac{\partial s}{\partial T}\right)_p T' + \cancel{\left(\frac{\partial s}{\partial P}\right)_T} P'$$

Sound wave

$$\mathbf{J} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n$$

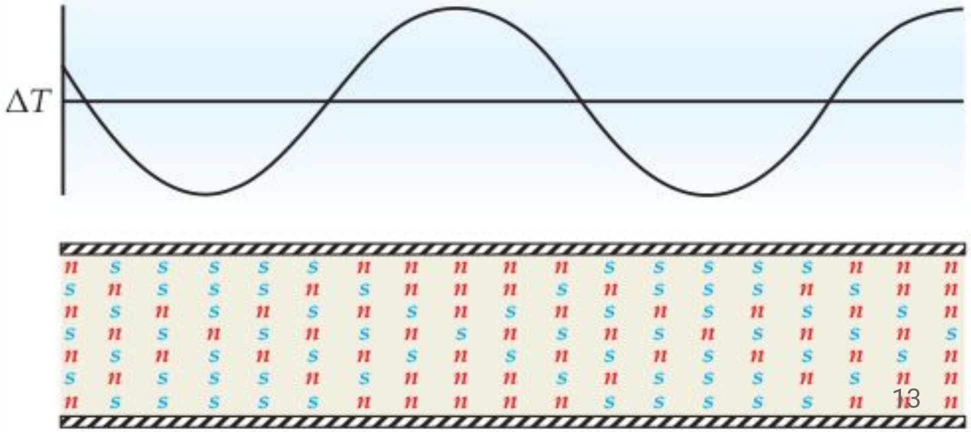
$$\mathbf{v}_s = \mathbf{v}_n$$



Entropy wave

$$\mathbf{J} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n$$
$$= 0$$

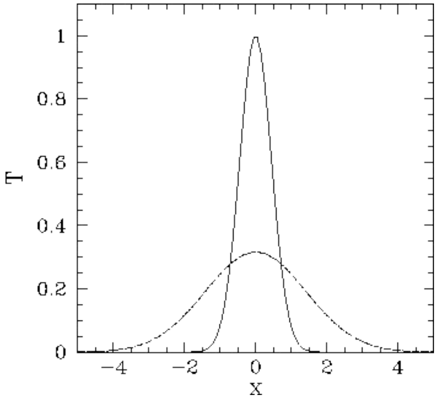
$$\mathbf{v}_n = -\frac{\rho_s}{\rho_n} \mathbf{v}_s$$



Diffusion VS Wave propagation

Diffusion

$$\frac{\partial T}{\partial t} = D\nabla^2 T$$

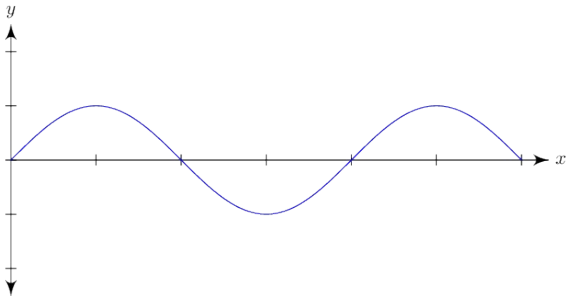


$$T \propto \frac{1}{\sqrt{t}} e^{-x^2/4Dt}$$

$$x_{front} \sim \sigma = 2\sqrt{Dt}$$

Wave-like

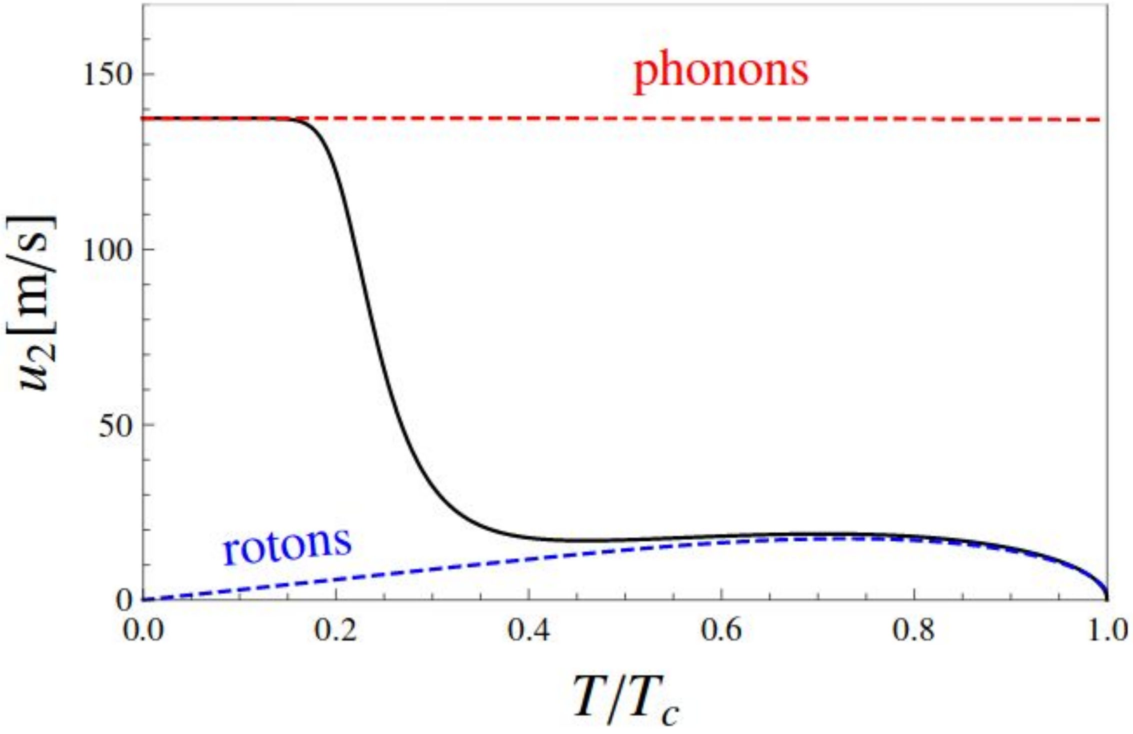
$$\frac{\partial^2 T}{\partial t^2} = u_2^2 \nabla^2 T$$



$$T \propto \cos[k(u_2 t - x)]$$

$$x_{front} = u_2 t$$

Speed of second sound



$$u_2 = \sqrt{\left(\frac{T_0 \rho_s s_0^2}{\rho_n c}\right)_p}$$

$$\rho_n = \rho_{ph}(T) + \rho_{rot}(T)$$

$$s_0 = s_{ph}(T) + s_{rot}(T)$$

$$\cdots$$

Conclusions

Superfluid Helium-4

- Superfluid transition at T=2.17K
- Peculiar properties such as viscosity and high conductivity
    - ➢ Two kind of “motions” for the fluid (two-fluid model)
    - ➢ Entropy/temperature waves