Quantum Computation with Spins and Excitons in Semiconductor Quantum Dots (Part I)

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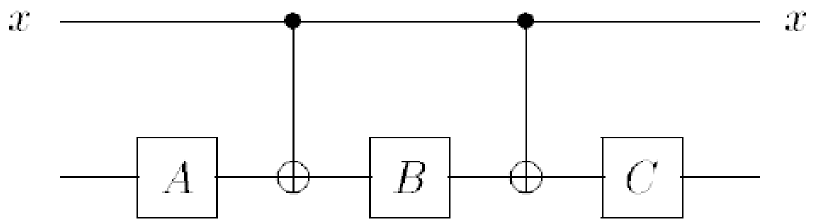
Dipartimento di Fisica, Pisa, Italy July 11th,14th, 15th 2008

Quantum Computing and Quantum Control Theory

Qubits and quantum gates



Control of a quantum system

$$H = H_0 + H_{control}(t, \sigma_1, \sigma_2, \ldots)$$

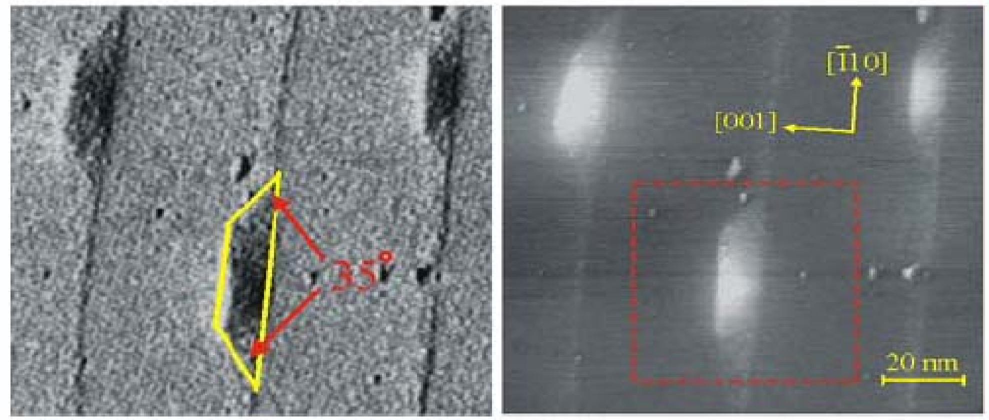
$$|\Psi\rangle = \alpha|\psi_0\rangle + \beta|\psi_1\rangle + \gamma|\psi_2\rangle + \delta|\psi_3\rangle + \ldots$$

Control Design and Optimization

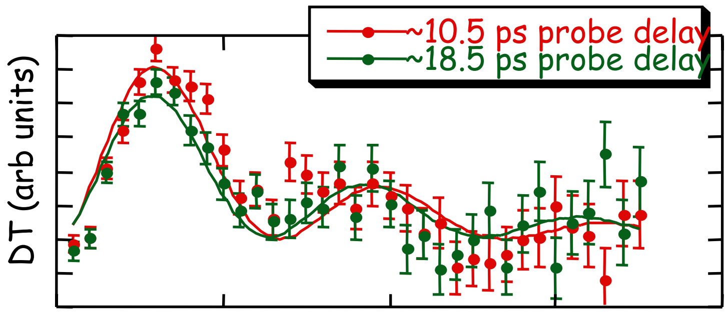
Quantum Dots

Optics

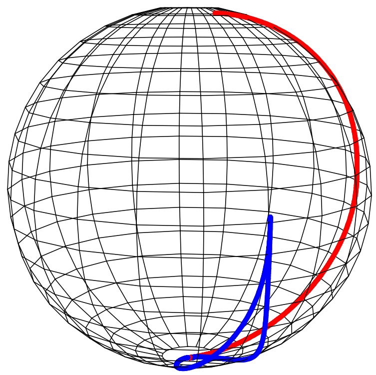
Today (Friday)



Quantum Dots



Optical Quantum Control of Excitons and Biexcitons in Quantum Dots

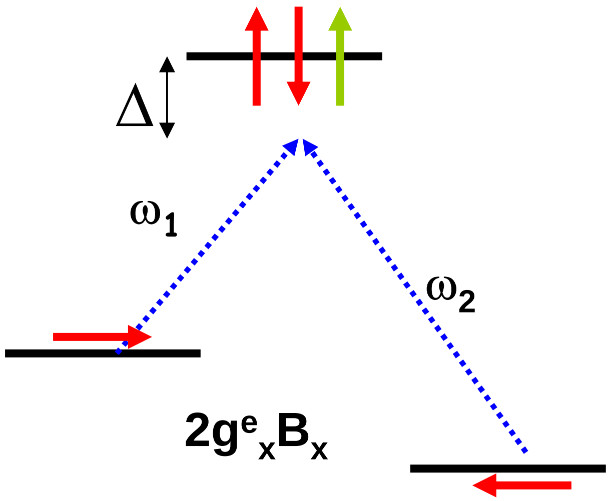


Two-Qubit problem in a QD

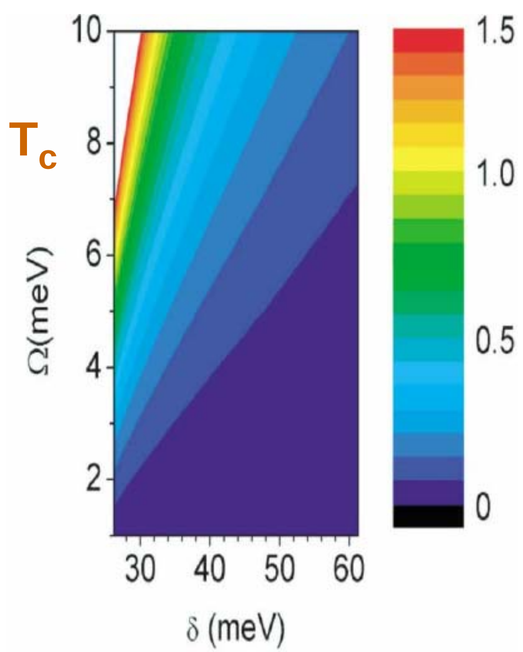
Pulse Shaping

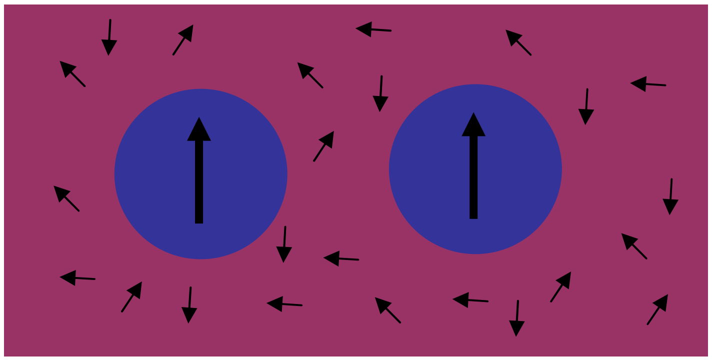
Monday

Optical Quantum Control
of Spins and Trions in Quantum Dots



Optical RKKY

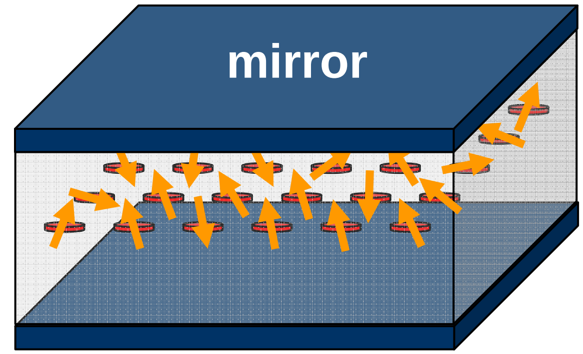


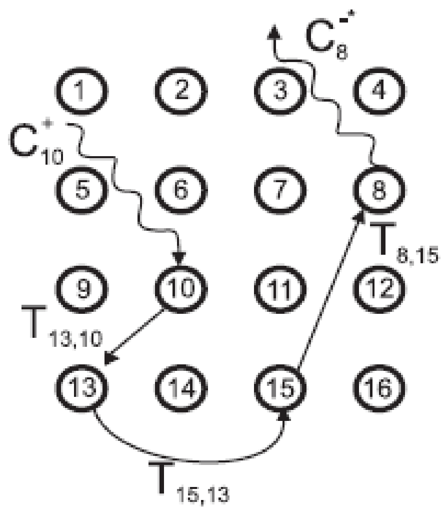


Optically Induced Ferromagnetism

Tuesday

Bragg Polaritons
Cavity QED and Spin



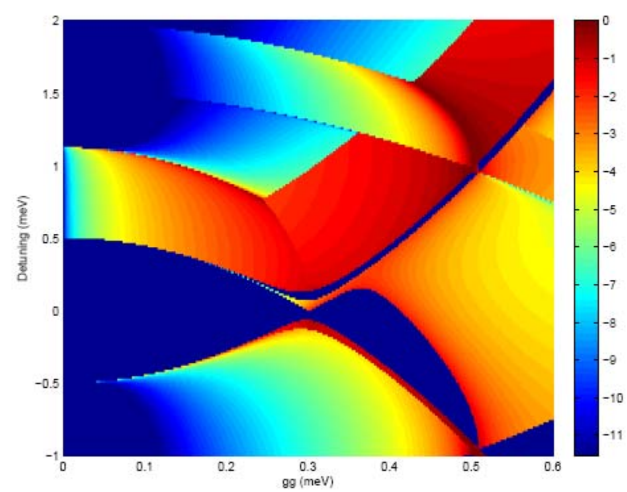


Multi-spin Interaction

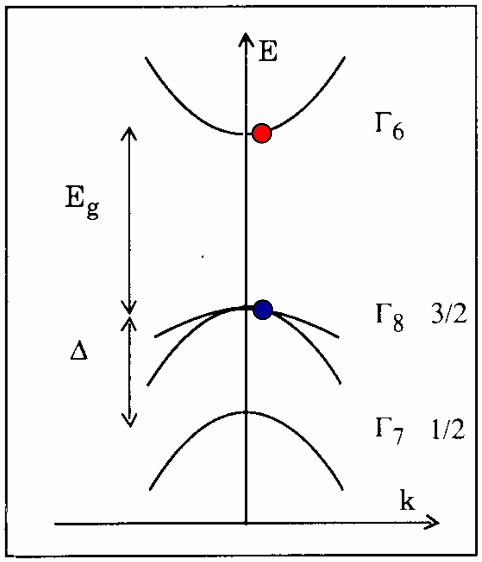
$$\hat{H}_T = \sum_{i>j} \tilde{J}^{(2)}_{ij} S_{iz} S_{jz} + \sum_{i>j>k>l} \tilde{J}^{(4)}_{ijkl} S_{iz} S_{jz} S_{kz} S_{lz} + \dots$$

Ground State
Multi-Spin-Entanglement

$$|GHZ\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle\right)$$



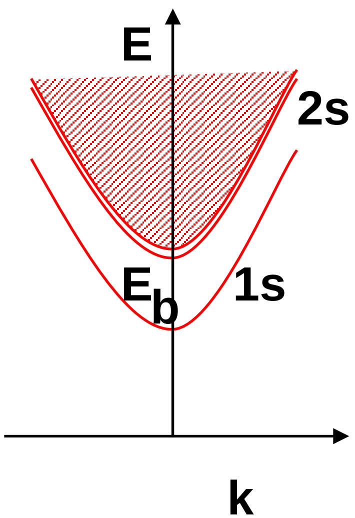
GaAs



Excitons are elementary optical excitations in semiconductors

The photo-excited electron and hole bind and propagate through the crystal (G. H. Wannier PR 37)

Hydrogen-like spectrum



$$a_B = \frac{\hbar^2 \varepsilon}{\mu^* e^2} \approx 80 \text{ Å}$$

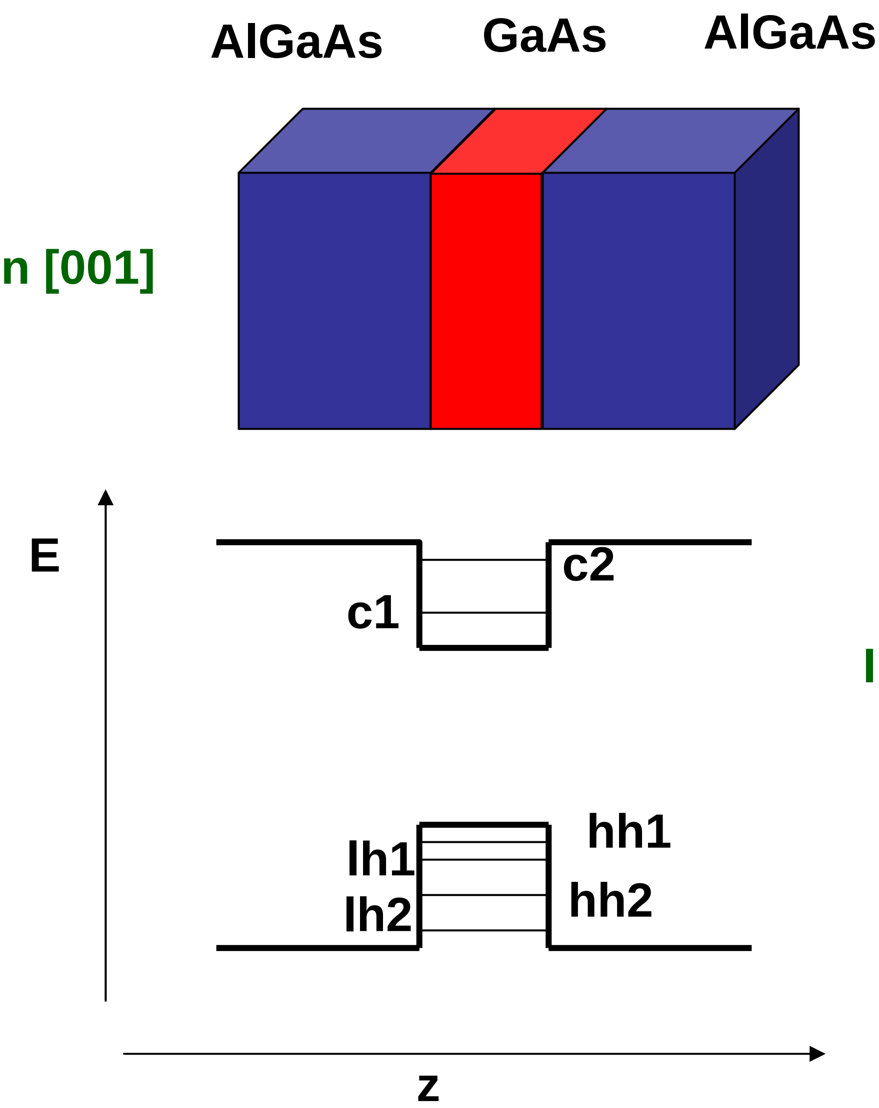
$$E_b = \frac{\mu^* e^4}{2\varepsilon^2 \hbar^2} \approx 5 \text{ meV}$$

$$\mu^* \approx 0.05\, m_0$$

$$\varepsilon \approx 13$$

Excitons confined in a quantum well

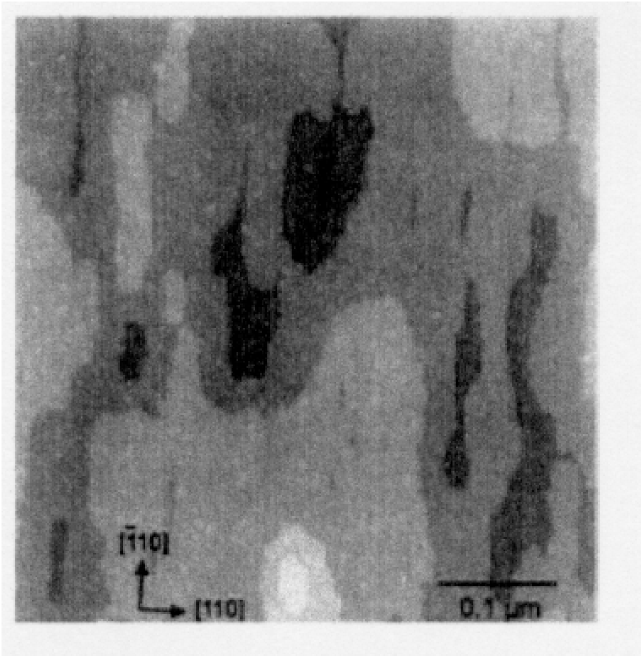
Growth direction [001]



In plane $k_{//}$ is a good quantum number

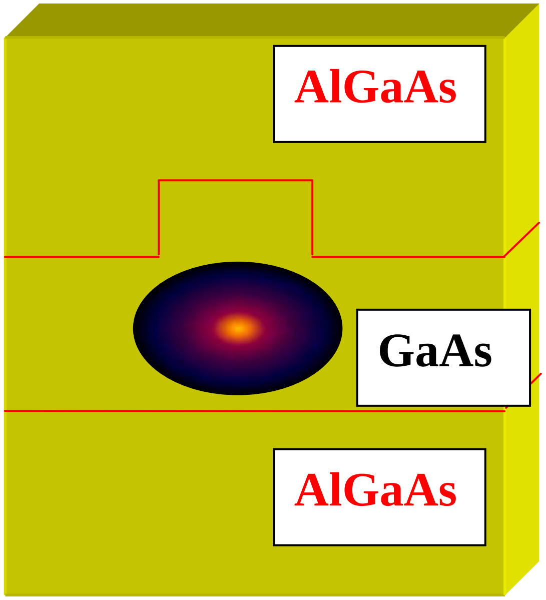
Propagation allowed only in the in-plane direction

Semiconductor Quantum Dots

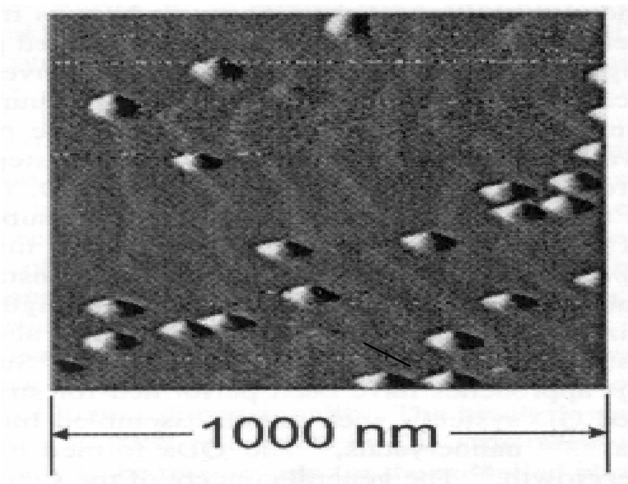


D.Gammon et al, Phys. Rev. Lett. 76,3005 (1996)

Interface fluctuation quantum dots (30 nm)



Quantum Dots

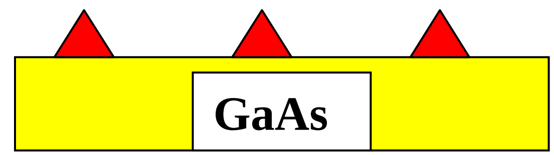


Strain-induced quantum dots (3 nm)

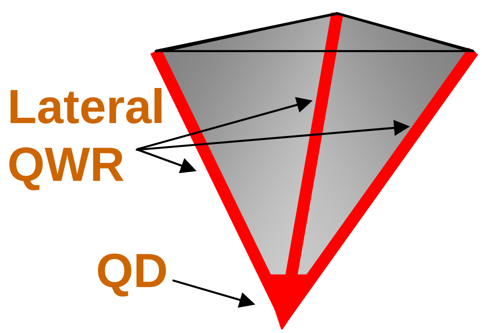
A. Zrenner *et al.*
J. Chem. Phys.
112, 7790 (2000)

Self assembled

InAs lattice mismatch

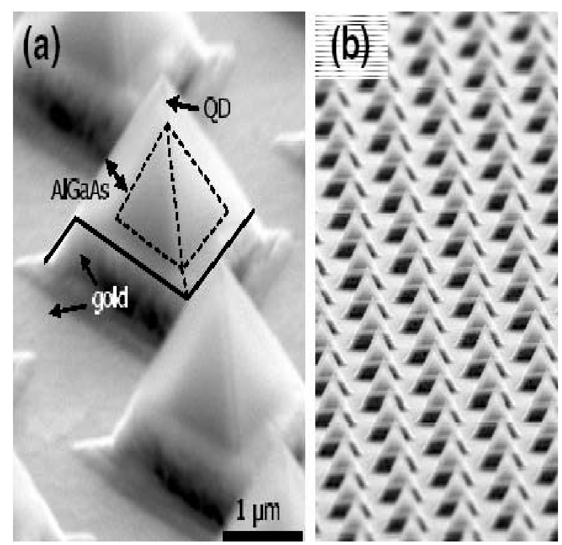


Pyramidal dots



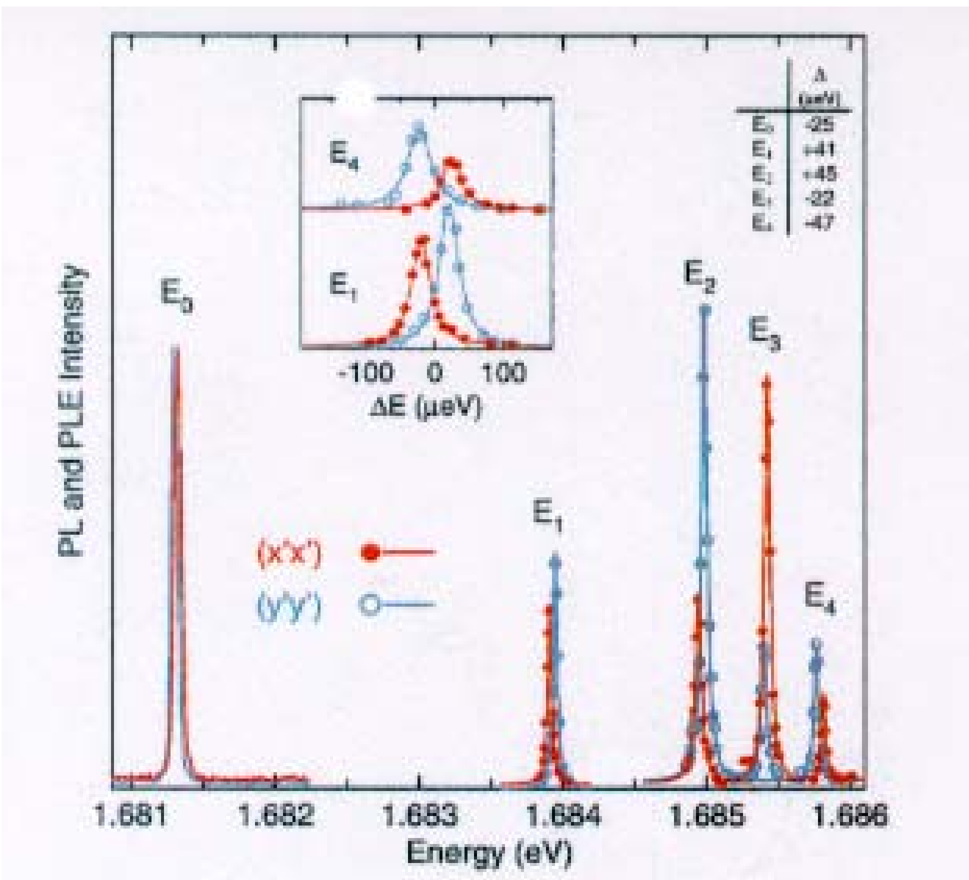
Self-limited growth (6 nm)

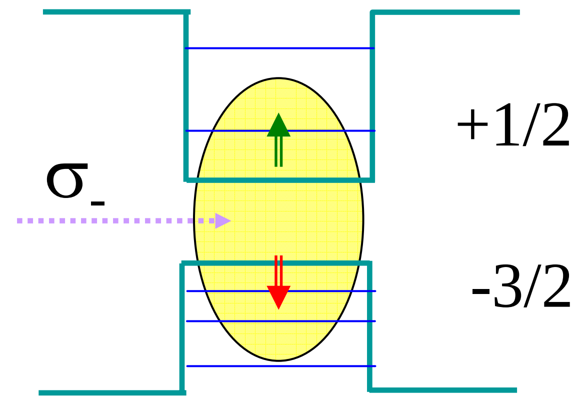
A. Hartmann *et al.*
J. Phys. Cond. Matt.
11, 5901 (1999)



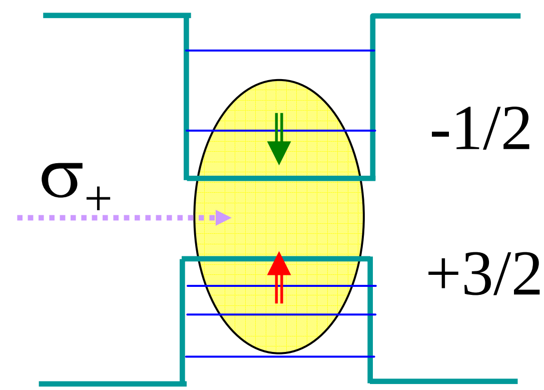
Excitons in a single QD

Interface fluctuation QD





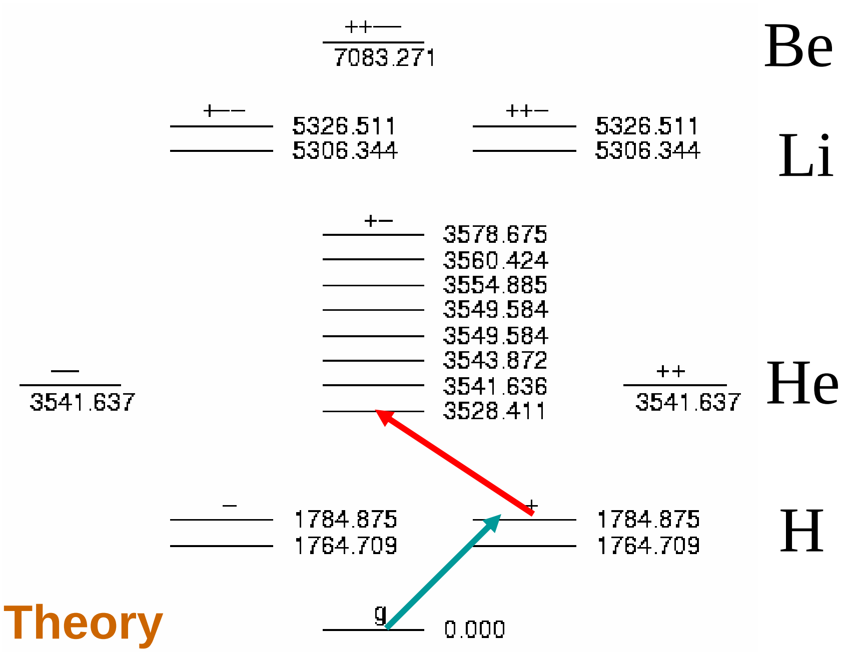
$X_{-}=e^{+}_{\uparrow}h^{+}_{\downarrow}|G>$



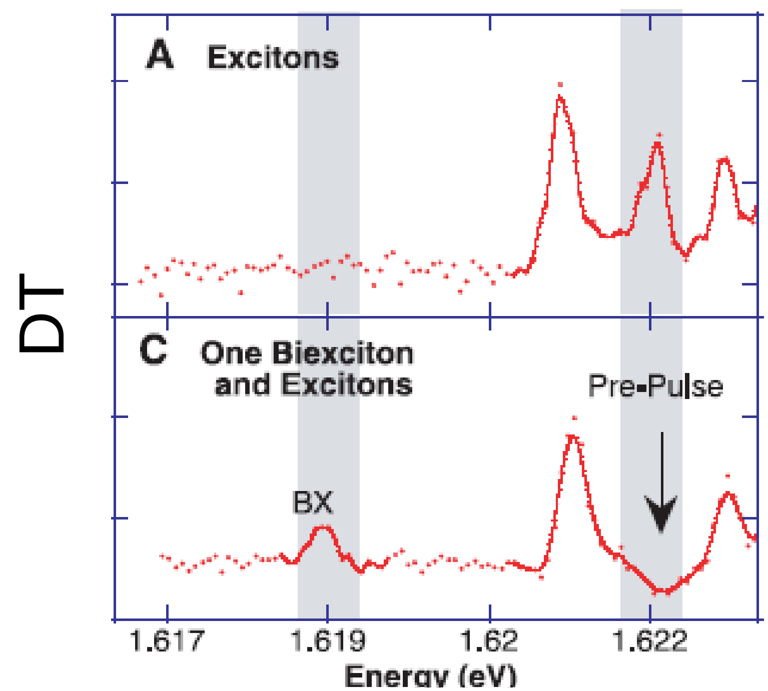
$X_{+}=e^{+}_{\downarrow}h^{+}_{\uparrow}|G>$

D. Gammon at al , Phys. Rev. Lett. 76,3005 (1996)

Multiexciton States



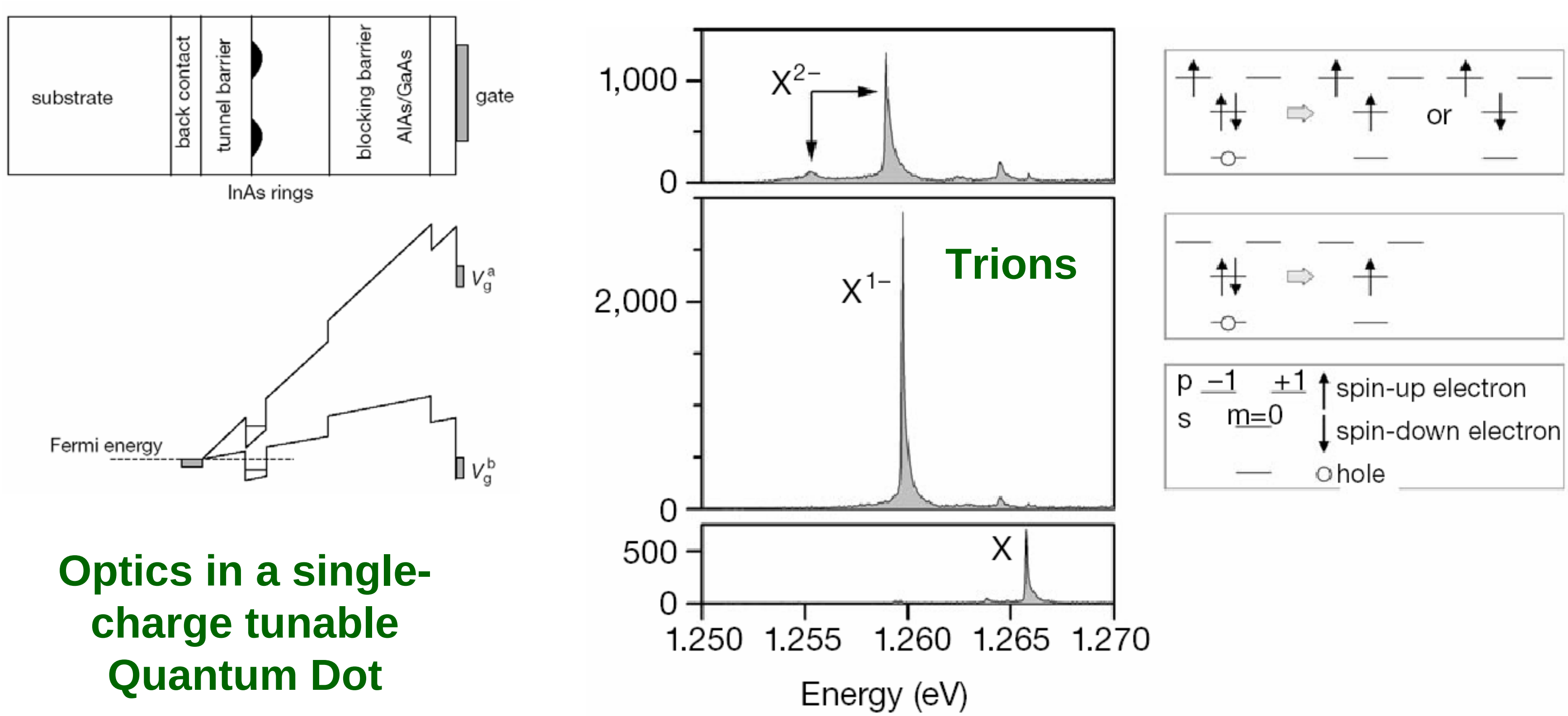
Theory



A QD is an "atom" where the number of protons and electrons can be controlled

X. Li et al Science 2003

Excitons in Charged Dots



Optics in a single-charge tunable Quantum Dot

Warburton *et al* ,
Nature 405, 926 (2000)

Optical Control

Direct Control of the Exciton and Biexciton state

$$\left|\Psi\right\rangle = \alpha\left|0\right\rangle + \beta\left|+\right\rangle + \gamma\left|-\right\rangle + \delta\left|-+\right\rangle$$

Rabi Rotations

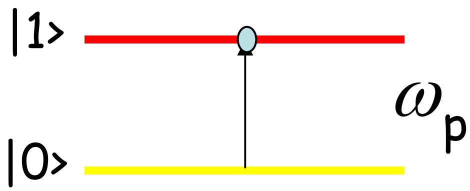
Light used for an indirect control of two spins

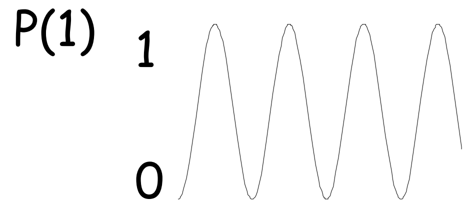
$$\left|\Psi\right\rangle = \alpha\left|\downarrow\downarrow\right\rangle + \beta\left|\downarrow\uparrow\right\rangle + \gamma\left|\uparrow\downarrow\right\rangle + \delta\left|\uparrow\uparrow\right\rangle$$

Adiabatic Raman Control, ORKKY

Rabi Oscillations

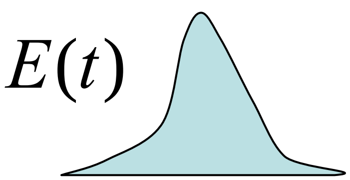
Two Level System





Rabi frequency

$$\Omega_R = \frac{d \cdot |E|}{\hbar}$$



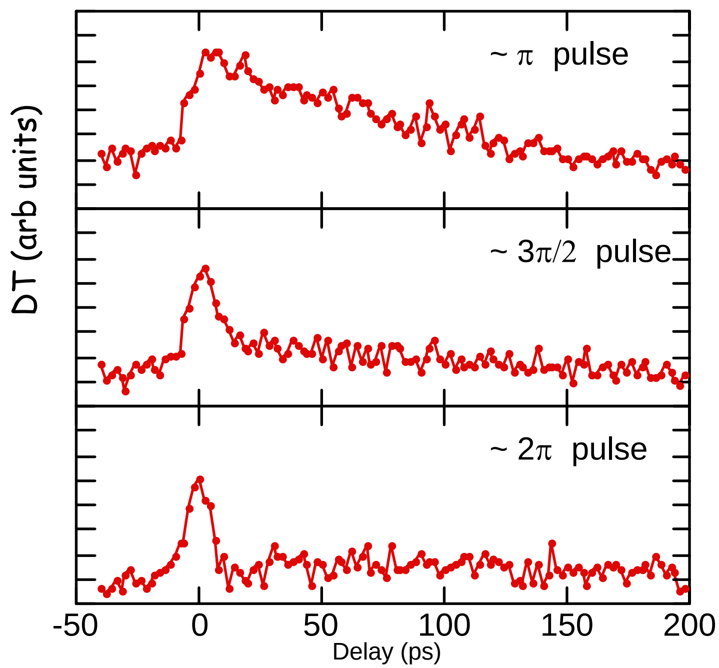
$$U = e^{-\frac{i}{\hbar}\int dt H(t)} = \begin{bmatrix} \cos(\frac{\beta}{2}) & -e^{i\alpha}\sin(\frac{\beta}{2}) \\ e^{-i\alpha}\sin(\frac{\beta}{2}) & \cos(\frac{\beta}{2}) \end{bmatrix}$$

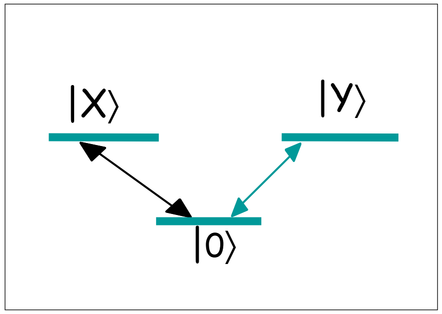
$$\beta = \int dt\ \Omega_R(t)$$

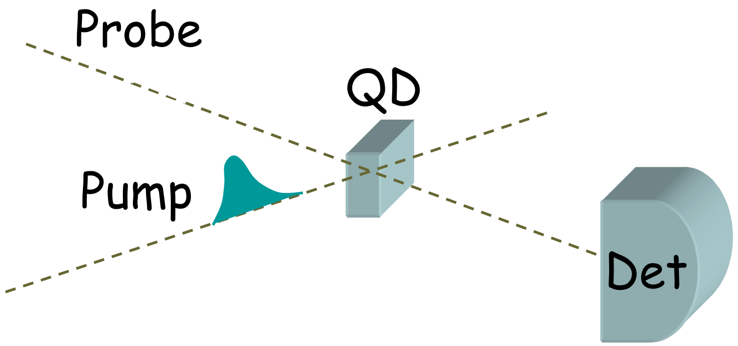
Optical Pulses give a full quantum control of the two level system

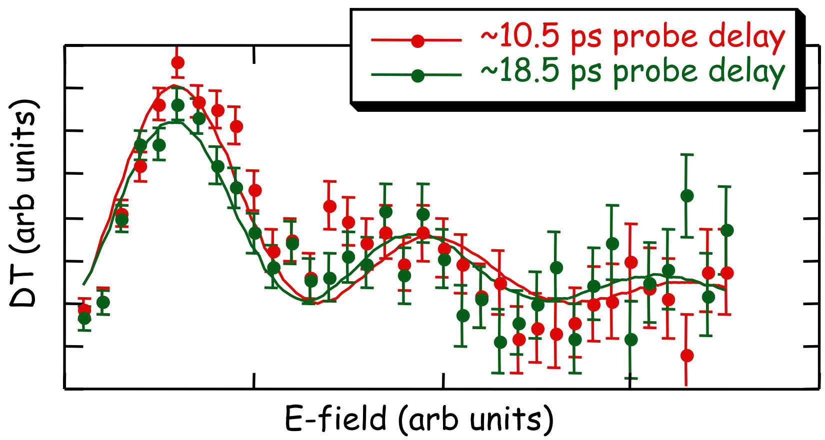
$$a|0\rangle + b|1\rangle$$

Rabi Oscillations of Excitons



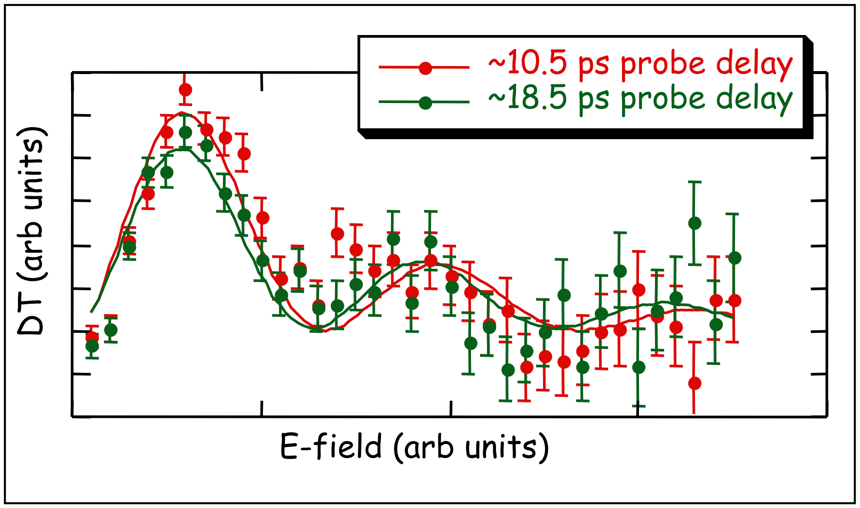


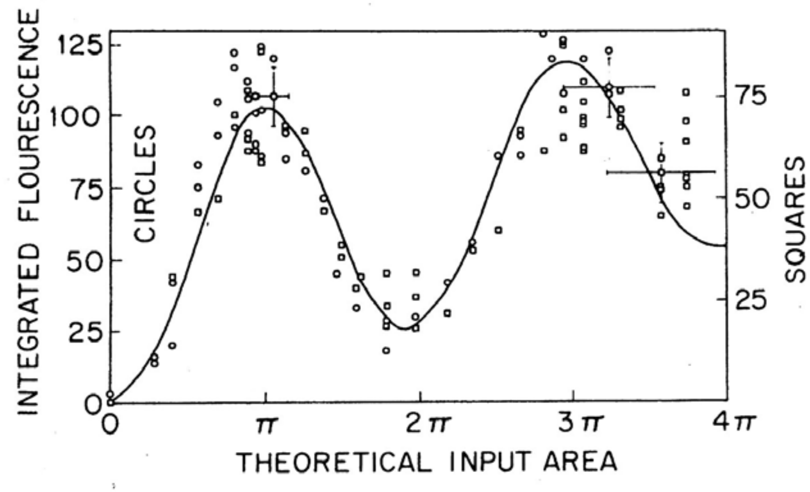




T. H. Stievater, X. Li, D. G. Steel, D. Gammon, D. S. Katzer, D. Park, C. Piermarocchi, and L. J. Sham, Phys. Rev. Lett. 87,133603 (2001).

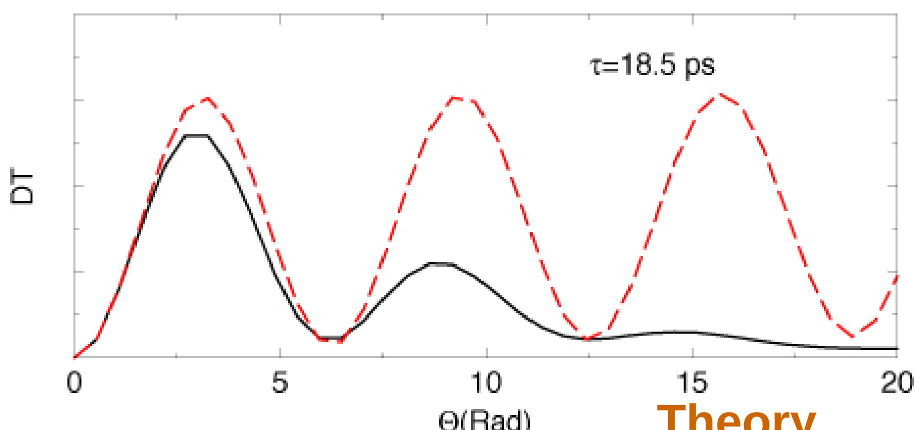
Excitons vs Atoms



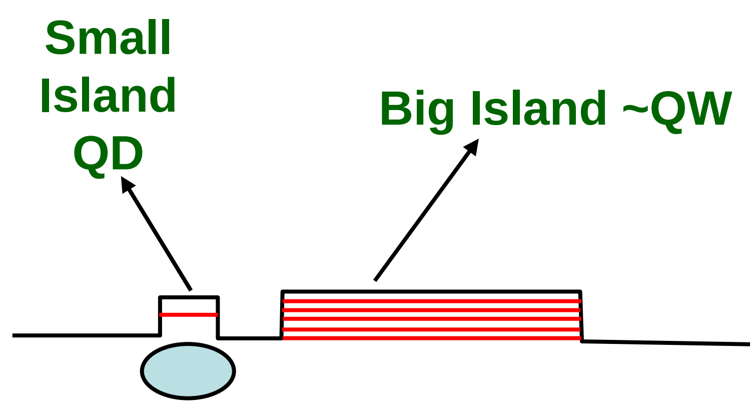


Atoms, H. Gibbs, (1973)

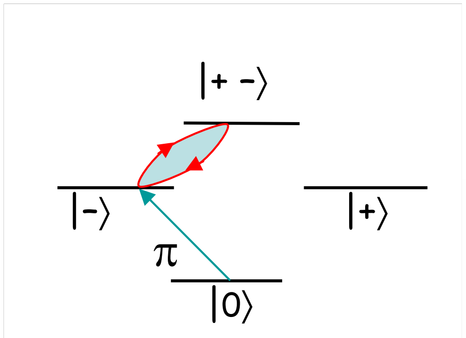
$$\frac{1}{T_1} = \gamma + \lambda n_{exciton}$$

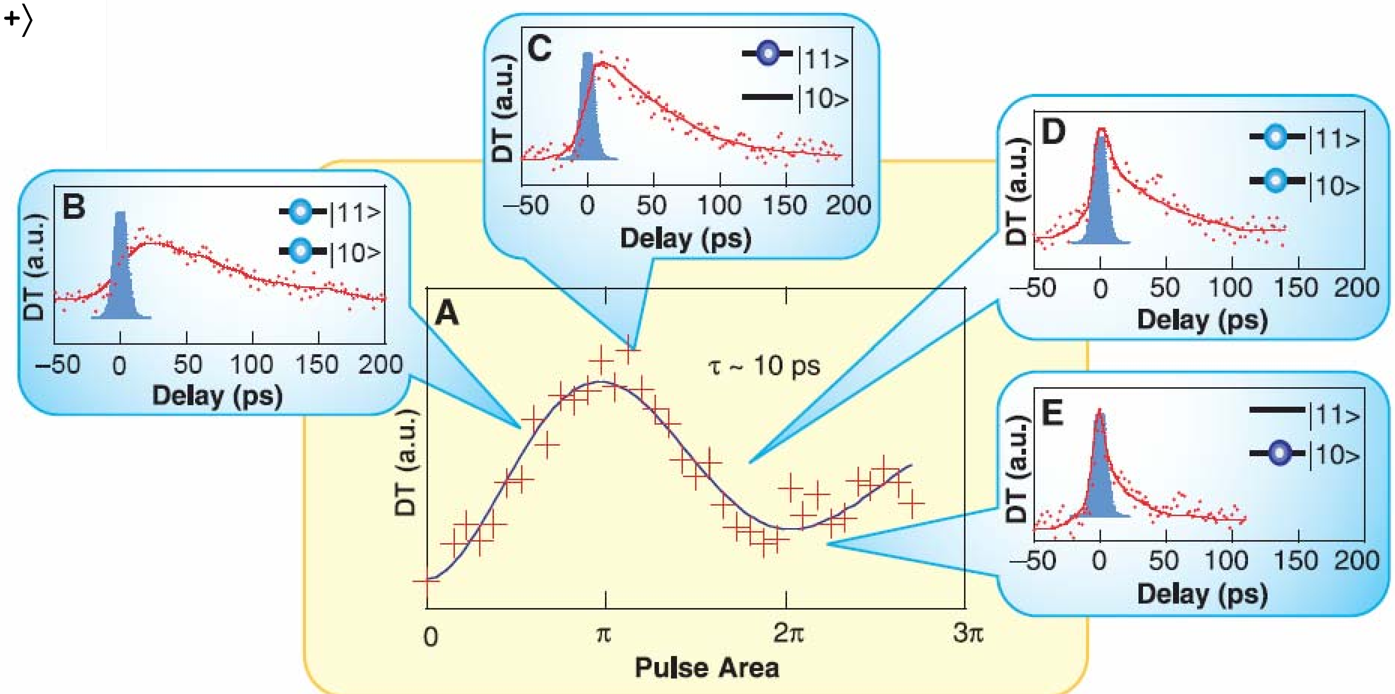


Theory



Control of Biexcitons





Ready to be used as a two-qubit quantum computer

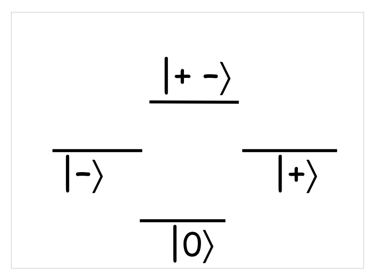
X. Li, Y. Wu, D. Steel D. Gammon, T.H. Stievater, D. S. Katzer, D. Park, C. Piermarocchi, and L. J. Sham, Science 301, 811 (2003)

Excitons are qubits

Qubit #1	1	$\|-\rangle$ **Exciton**	**Single qubit #1 rotation are provided by pulses** σ_-
	0	$\|0\rangle$	
Qubit #2	1	$\|+\rangle$ **Exciton**	**Single qubit #2 rotation are provided by pulses** σ_+
	0	$\|0\rangle$	

$$\boxed{H_1 \otimes H_2}$$

$$\begin{array}{cc} |11\rangle & |+-\rangle \\ |10\rangle & |+\rangle \\ |01\rangle & |-\rangle \\ |00\rangle & |0\rangle \end{array}$$

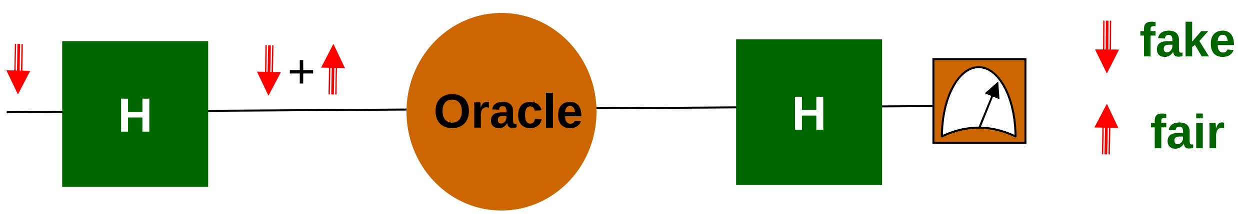


The Deutsch-Jozsa Parlor Game

4 different coins: two fair and two fake

An Oracle looks at one side of a coin and tells you if it is head or tail

How many consultation of the Oracle do you need to find out if a coin is fair or fake?



Using quantum superposition and quantum interference only one consultation is needed

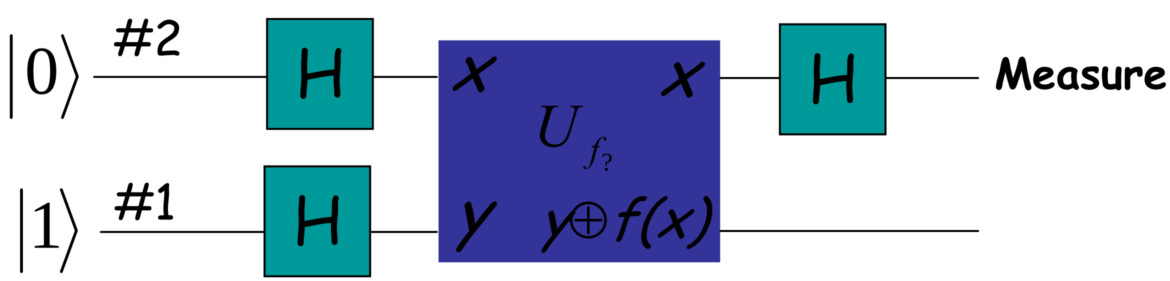
Deutsch problem

$f_?(x)$ constant or balanced?

Classical computing: I have to evaluate both

$f_?(0)$ $f_?(1)$ **and compare the results.**

Quantum computing: build a Unitary transformation associated to $f_?$ acting on two qubits



➡ 1 **Balanced**

➡ 0 **Constant**

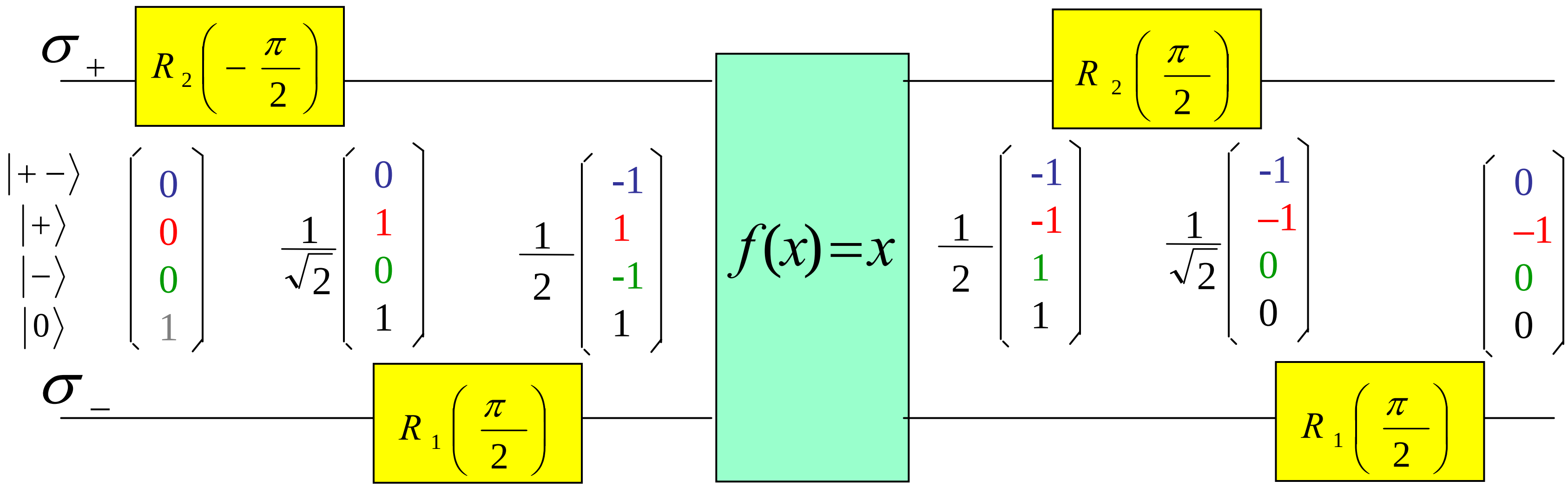
$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

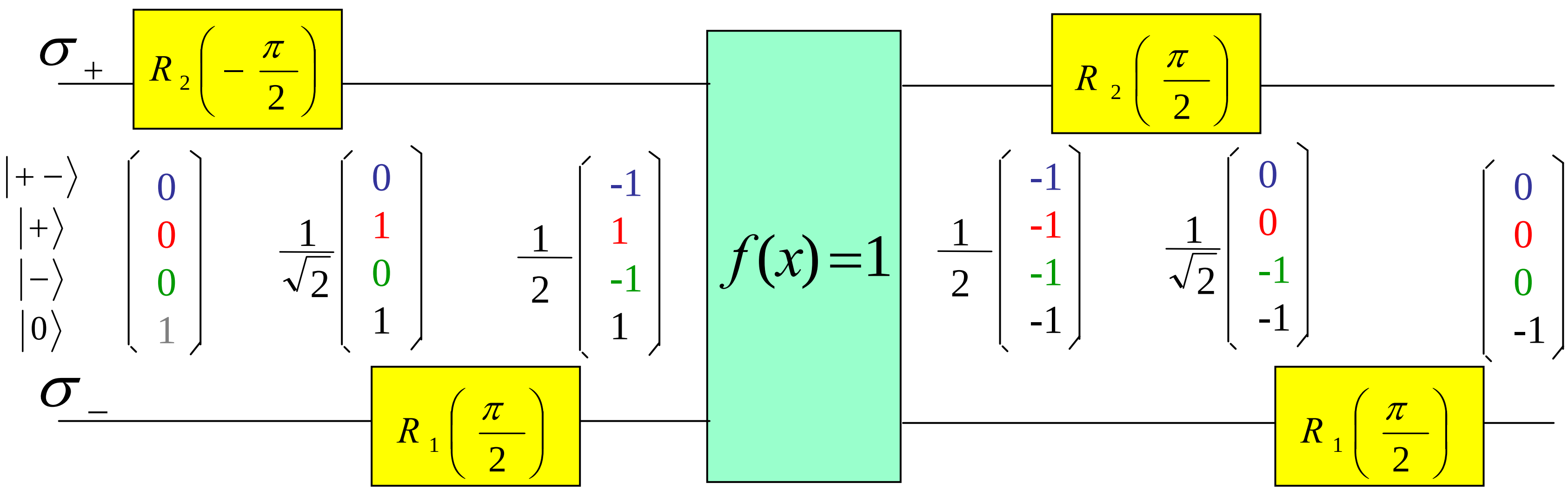
We get the answer in one shot !

$$U_{f_?}\left|\overset{2}{m},\overset{1}{n}\right\rangle = R_1^{f_?(m)}(\pi)\,|m,n\rangle$$

$R_1^1(\pi)$ Rotates +π

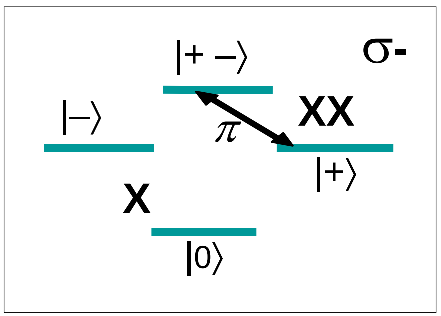
$R_1^0(\pi)$ Rotates -π

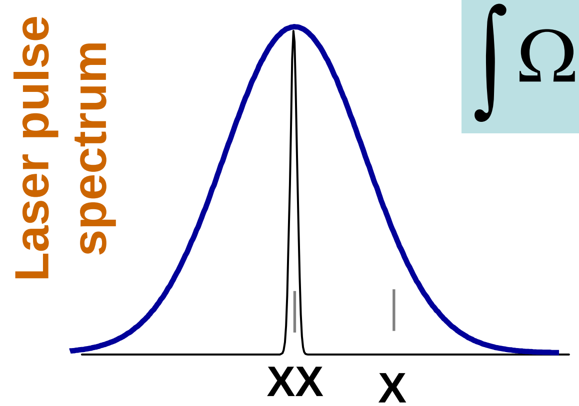




Optimization of the design:
Pulse Shaping

Double two-level system





$$\int \Omega_R d\tau = \pi$$

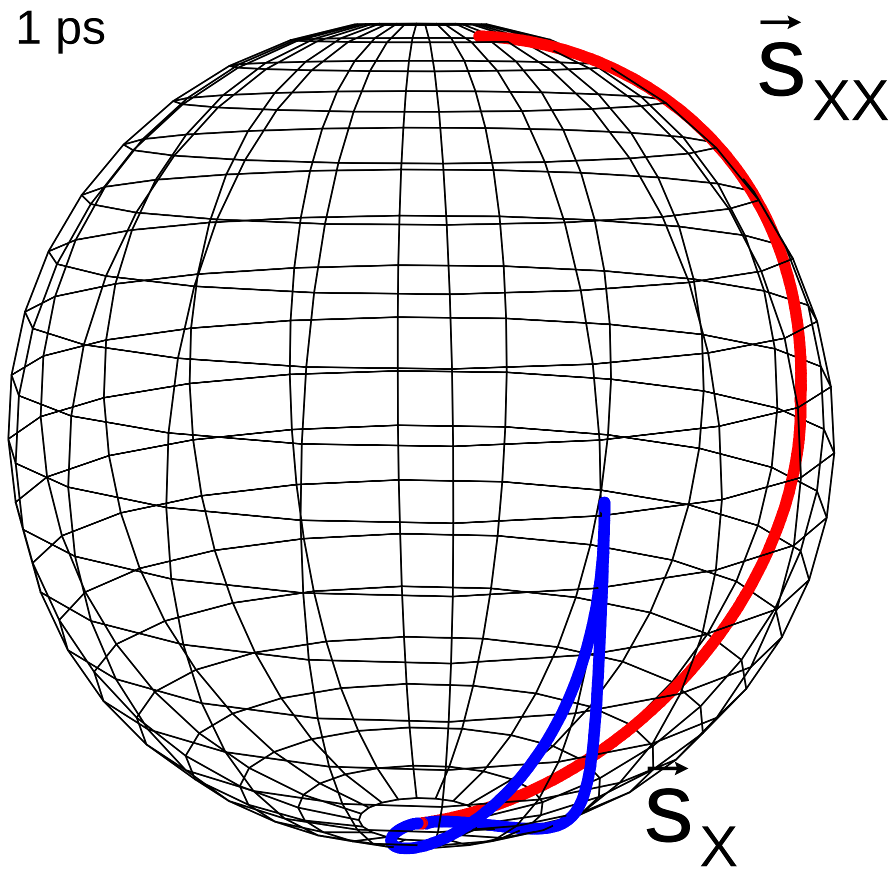
Resonant Excitation ⟺ **Long Pulses**

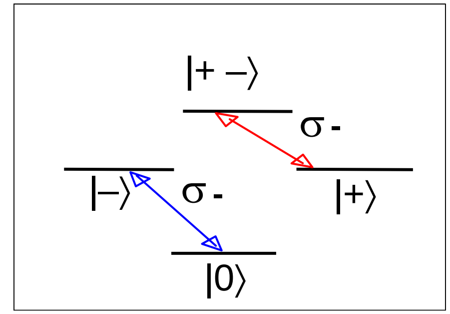
Dephasing (~ 40 ps)

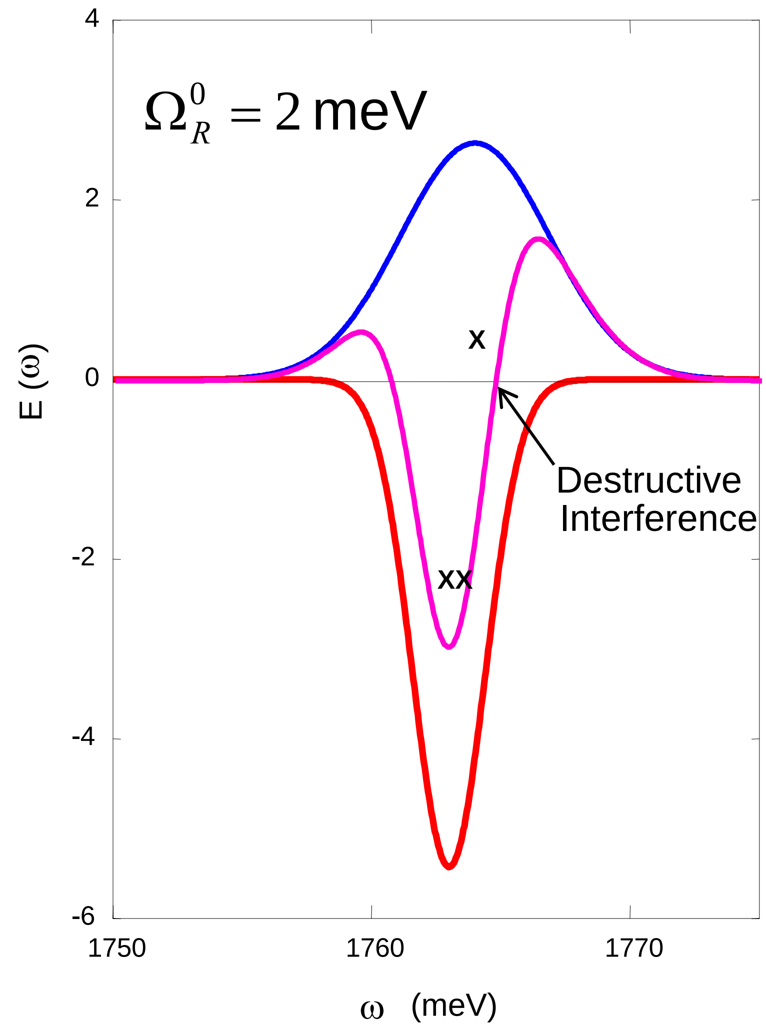
Short time for the operation

Two phase-locked pulses:

$$\Omega_R(t) = \Omega_R^0 \left(e^{-(t/s)^2} e^{-i\omega_X t} + e^{-(t/s1)^2} e^{-i\omega_{XX} t - i\pi}\right)$$







Fidelity: $\overline{\left|\left\langle \psi_{in} \,|\, \tilde{U}^{+} U_{ideal} \,|\, \psi_{in} \right\rangle\right|^{2}}$

F=0.535 without shaping

F=0.995 with shaping

Analytical Methods

$$H_{control}(t, s, s_1, \Omega_0)$$

Numerical maximization of the Fidelity

Magnus expansion

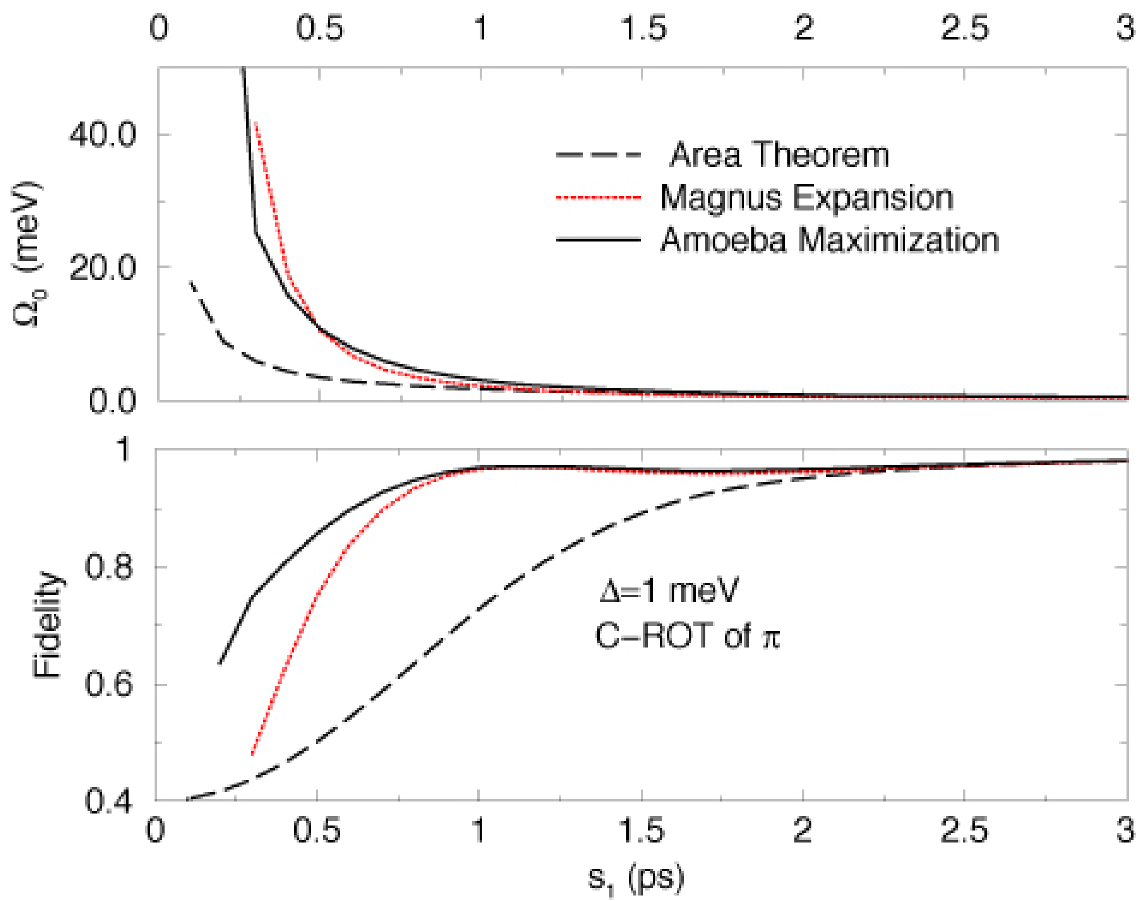
$$U = Te^{-\frac{i}{\hbar}\int_0^{\infty} H_C(t)dt} = e^{-\frac{i}{\hbar}(S^1{}_C + S^2{}_C + S^3{}_C + ...)}$$

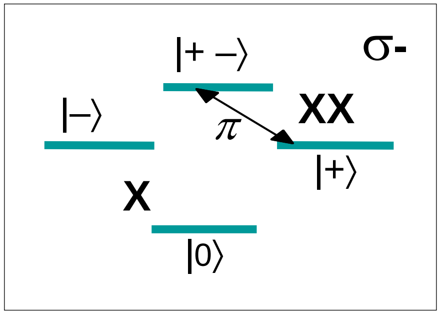
$$S_C^1 = \frac{1}{2}\int_0^{\infty} H_C(t)dt \qquad S_C^2 = \frac{-i}{8}\int_0^{\infty} dt \int_0^{t} dt' \left[H_C(t), H_C(t')\right]$$

For a given U is possible to find an analytical expressions for the control parameters

C. Piermarocchi , P. Chen, and L. J. Sham, PRB (2002)

C-ROT





Magnus expansion

$$s = s_1 e^{-(\Delta s_1/2)^2}$$

$$\Omega_0 = \sqrt{\pi}\left(s_1 - s e^{-(\Delta s/2)^2}\right)$$

Simulation of Deutsch in a QD

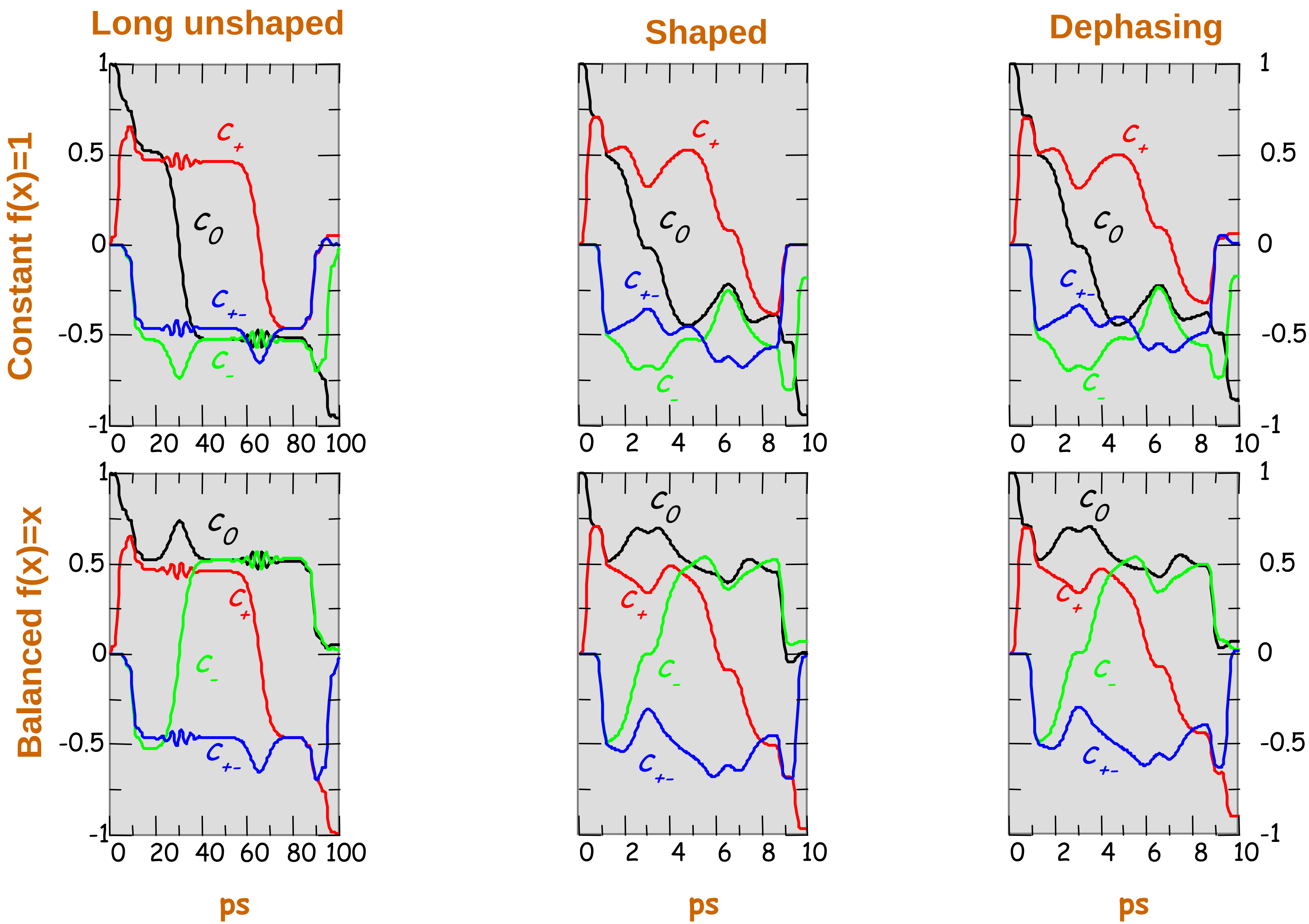
Six multi-exciton states with 4 level for 2 qubits. (Other states are sufficiently far away.)

Decoherence included

Pulse shaping

Pochung Chen, C. Piermarocchi, L. J. Sham, *Control of Spin Dynamics of Excitons in Nanodots for Quantum Operations*, Phys. Rev. Lett. 87, 067401 (2001).

Time Evolution



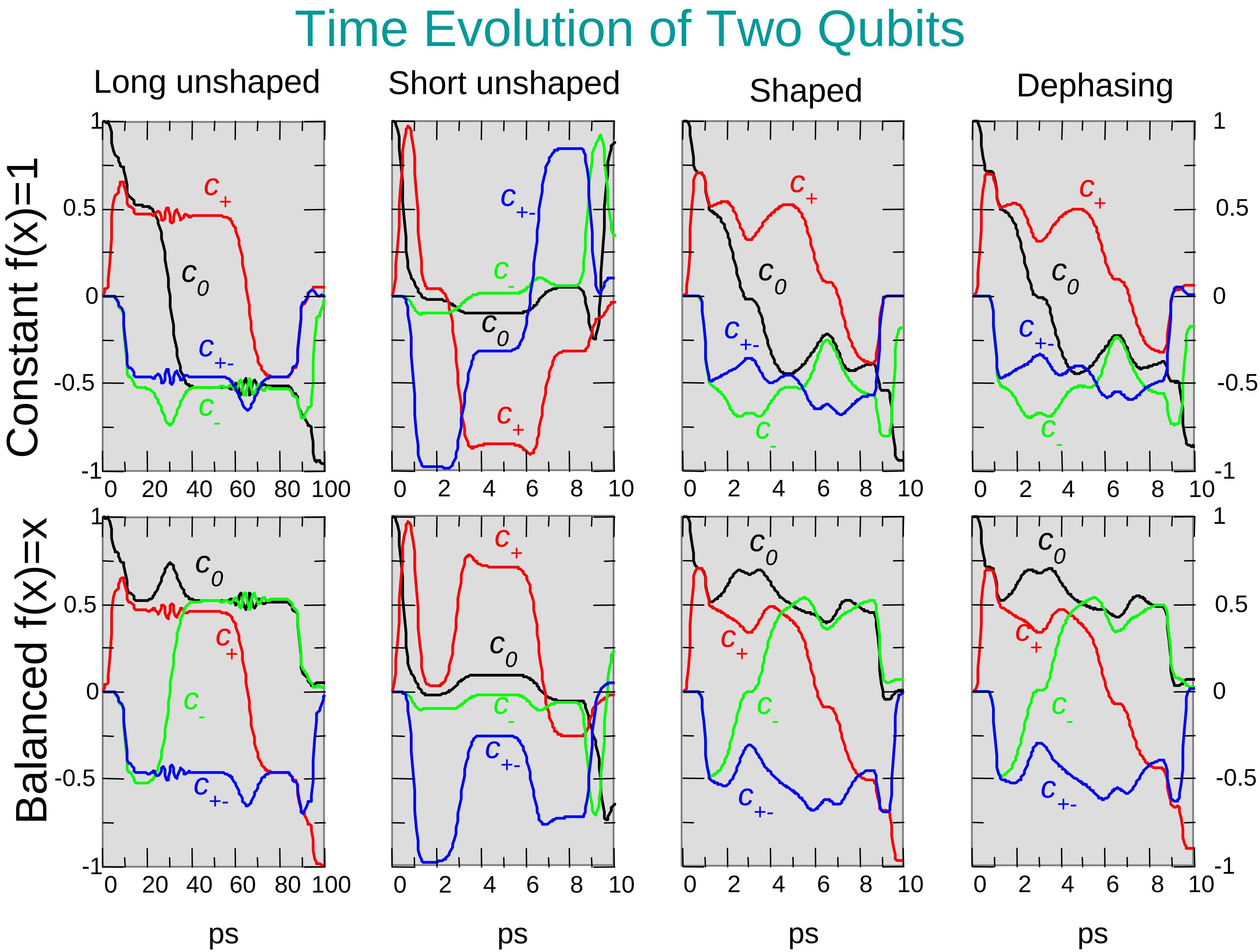
Conclusions

Control of exciton and biexciton in a QD

Experimentally realized

Readily applicable to two-qubit quantum algorithms

Benchmark for issues in the optical control design



Time Evolution of Two Qubits
Long unshaped
Short unshaped
Shaped
Dephasing
Constant f(x)=1
Balanced f(x)=x
c_+
c_0
c_{+-}
c_-
ps